

GENERAL DISCUSSION

DONALD M. BURMISTER.¹—In this discussion consideration is given to why basic differences in consolidation testing methods should be necessary in the cases described by the authors and what fundamental principles are applicable, as a basic framework for consolidation testing methods. The concepts and principles of the controlled test method were presented and discussed under this heading in the writer's paper.

In soil and foundation work it should be recognized that the physical laws governing the inelastic behavior of soils under stress are complex and obscure, and are imperfectly known and comprehended at present with no simplicity or uniformity of action to be expected. Hence there may be important elements of uncertainty, first regarding the existence and real nature of the controlling conditions in situations and their relative dominance and influence on soil behavior, and second, regarding how they can be translated successfully or reasonably closely into terms of basic test conditions and principles of soil testing for each individual case. The failure, however, to recognize, to comprehend, and to take into account one potential behavior characteristic or one controlling condition, or the failure to estimate properly their relative dominance in a situation, may introduce uncertainties and difficulties into a situation of which the engineer may be quite unaware. He then may never fully understand why

soils behaved erratically or differently from what was expected. Adequate information and accurate analyses are an insurance against unforeseen difficulties, and reduce the hazards and uncertainties inherent in soil and foundation work by directing attention specifically to those conditions and difficulties in a situation for which proper provisions can then be made in advance.

A complete visualization and appraisal of each situation is essential and is of inestimable value. But in order to recognize and to comprehend a situation in its entirety the soil engineer must crystallize his thoughts and judgments and must formulate them in logical order into *words*. Such an analysis should contain: (1) a summary of all available exploratory field data, of laboratory tests data and test curves, and of foundation plans of structures; (2) an accurate and complete interpretation and appraisal of a situation with regard to the controlling conditions and their relative dominance; (3) a statement of the test conditions adopted in the soil testing program for the particular project with reasons therefore; (4) a case history of the project, if such information is obtainable, at each stage of construction; and (5) a comparison of the predictions with the actual observed behavior of the soils with such evaluations as possible of the reasons for any discrepancies. Such an analysis would increase a person's power to recognize, to comprehend and to appraise, making it less likely that he will fail to take into account any im-

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portant element in a situation. A serious lack of some essential information may be brought to light, in which case such additional information can be obtained before it is too late. Such an analysis makes it possible to evaluate more accurately the reasons for any discrepancies between the predicted and observed soil behavior. Finally it enables a person to build up for himself a reliable background of knowledge and experience, to increase his capacity for analysis and interpretation, and to improve his powers of discernment and of judgment for handling present and future soil and foundation problems. Brief summaries of such analyses are given for the two cases cited in the writer's paper.

The purpose of the discussion of the papers of the symposium is to give point to these concepts and possibilities of controlled test methods by pointing out a few of the dominating controlling conditions and the corresponding test conditions that were introduced, or in the opinion of the writer should have been specifically introduced in the consolidation techniques used in the analysis of the problems presented by the authors of the papers.

The paper by Messrs. Thompson and Palmer is of special interest, because it deals with the use of peat as a foundation material, which usually is considered to be without any allowable supporting value. The most important controlling conditions in the situation are: the great thickness of the peat and peaty clay-soil of some 29 and 20 ft, respectively, the high moisture contents of 240 to 340 per cent and the low degree of consolidation with unit dry weights of 14.5 to 20.8 lb per cu ft; a very high compressibility, a short primary consolidation period due to the open porous nature of peat and an exceptionally long-time secondary consolidation of important magnitude

due probably to internal plastic creep and yielding of the fibrous structure of peat; the relatively heavy loading by the earth fill and the degree of consolidation attained under the overloading and the subsequent construction of the earth-filled concrete barricades several months later.

The special test conditions definitely introduced in the consolidation testing was the long consolidation time interval allowed under the design loading of 11 to 36 days in order to determine the magnitude and time-rate of secondary consolidation, which was basic information required in this case. Because of the very compressible nature of peat and of the marked disturbance effects of large load increments, it is believed that the initial stress should have been not larger than $\frac{1}{16}$ ton per sq ft and that the increments of stress after 1 ton per sq ft should have been in a ratio of 0.5 instead of the usual ratio of 1.0. In the writer's experience no difficulty has then been encountered in determining the natural initial stress for such materials. This information is essential for predicting the additional settlement to be expected beyond that already attained.

The paper by Messrs. Holtz and Gibbs is also of special interest because it deals with a foundation material having very objectionable settlement characteristics for hydraulic structures, where flooding of the foundation soils is a definite possibility. The more important controlling conditions in the situation are: the thickness of the loess deposit, the exceptionally loose natural compactness and open structure, the low percentage of natural saturation, and the rather exceptional natural coherence of the undisturbed material in the low moisture state due to clay films at the grain contacts; the relatively low compressibility under low consolidation stresses at natural mois-

ture content, the relatively marked increase in compressibility above the equivalent natural initial stress associated with a breakdown of structure, and the exceptionally high compressibility either wetted under stress or prewetted, due to softening and elimination of the natural bonds; hydraulic type of structure, character of loading, and limitations on the permissible total and differential settlements; and either the definite condition of flooding of the foundation soils, or of rise of ground water to cause objectionable settlements after construction; the possibilities of definitely improving foundation conditions either by prior treatment or by treatment during and subsequent to construction.

The special test conditions definitely introduced in the consolidation testing for this particular study were: (1) consolidation at natural moisture without flooding of the specimens to some specified stress, followed by flooding of the specimen, and (2) consolidation on prewetted specimens in order to determine the comparative consolidation-settlement-time relations under such conditions for different design stresses. It is evident that these test conditions must be introduced into consolidation or triaxial compression testing, where an appraisal of the controlling conditions in a situation show definitely that flooding must be taken into account, or that the ground water may rise to objectionable heights. It must be emphasized, however, that if an appraisal of the controlling conditions in a particular situation in regions of loessial soils or of soils possessing similar consolidation characteristics shows that the possibility of such flooding or rise of ground water are very remote, then the soils in the consolidation test should be tested throughout at natural moisture with no flooding of the

specimens, but with moisture loss prevented by suitable means, as the basic test condition. This was the essential test condition used in the consolidation testing described in Case 1 of the writer's paper in order to maintain the natural coherence in a fully active and effective state. And as a matter of fact and of interest, the use of flooding and of hosing to compact the backfill during construction was ruled out in favor of more expensive and slower compaction methods because of the possibility of objectionable effects on the natural structure and coherence of the sand deposit directly beneath the foundation of the structure.

The paper by Messrs. Shockley and Mansur is also of special interest, because it deals with the notably poor and difficult foundation conditions encountered in the deep and variable alluvial deposits of the Lower Mississippi River. The more important controlling conditions in the situation are: the great combined thickness of the three principal compressible strata, the relatively more rigid and less compressible character and load-spreading capacity of the upper stratum 1 of some 10 to 15 ft in thickness; the compressible character of the clay-soils of strata 2 and 3, and the relatively low permeability and long-time consolidation delay characteristics, a state of overconsolidation in stratum 1 with a natural initial stress exceeding the overburden stress, and a state of probable normal consolidation in the underlying strata 2 and 3: a height of compacted embankment of 24 to 30 ft with a rather great width, as affecting the magnitude and depth of penetration of consolidation stresses.

It has been the writer's practice in such cases to establish as closely as possible the state of consolidation and to re-establish upon the consolidation test specimen the natural initial stress at

each depth by the test condition of an unloading-reloading stress cycle, as discussed in Case 2. In the state of over-consolidation the very flat reloading stress-cycle curve is used in settlement and time-consolidation analyses, starting from the overburden stress. In a state of natural consolidation the primary consolidation curve is used for stresses imposed above the natural initial stress. The time-delay phenomena in a state of over-consolidation is much shorter. This fact together with the higher intensity of stresses imposed in the upper stratum 1 may account for the more rapid rate of settlement observed in the early stages of consolidation than predicted.

Messrs. Matlock and Dawson describe a useful method for consolidation research testing, which should become an accurate and valuable research tool, particularly in the development, investigation, and evaluation of testing equipment and test procedures, so as to achieve reproducibility of test results and an elimination of the personal element. The special test condition introduced was the complete remolding of clay-soils and the use of de-aired, seasoned, and extruded samples, by which a high degree of uniformity of moisture content, consistency, and structure could be achieved. Some of the spread in the consolidation curves may have been due to differences in the completeness of remolding of the material in the balloon just prior to extrusion of a specimen with the result that different degrees of structure actually existed, which remained from that built up by thixotropic effects during the seasoning of the material.

Mr. Karol in his paper has proposed a rapid technique of consolidation testing, which may serve a useful purpose for general classification purposes for comparing different soils regionally with regard to their consolidation and shearing

strength characteristics. However, for specific consolidation problems, accelerated consolidation testing techniques cannot be employed, because of the enormous importance of the strain-rate and time-consolidation effects upon consolidation and shearing phenomena, as pointed out in the writer's paper. The argument that the time required for a consolidation test is inconsistent with the accuracy of the test data is not valid, because it is the time-delay phenomena that must be taken into account. Rather the consolidation test procedures must be refined to produce better agreement between the predicted and observed phenomena. This is the purpose of the controlled test methods and of an accurate and complete appraisal of the controlling conditions that dominate in each particular situation, as illustrated in the two cases cited.

Mr. Finn, in his paper has called attention to a most important element in consolidation testing, that has been given very little consideration, namely the effect of temperature on the consolidation phenomena. In this study it was found that temperature primarily affected the time-rate of consolidation in the case of remolded materials. There is some indication in Fig. 6 that the temperature affects the stress-strain curve as well with a parallel vertical displacement downward under lower temperatures. It has been the writer's experience, however, in testing undisturbed clay-soils, that appreciable temperature variations affect the slope of the pressure-voids ratio curve, sometimes to make the test results useless, making the slope flatter for temperature drops and steeper for temperature rises. It is believed that the nonrecoverable displacements, which are a large proportion of the total consolidation, should be most affected in magnitude by temperature effects, since they are of a plasto-elastic nature. Therefore the

writer has become convinced that the essential test condition should be introduced in the consol-testing of undisturbed samples of making the temperature throughout the test constant at approximately the natural ground-water temperature.

The paper by Mr. Barber is of interest because it deals with a number of problems of structures founded on the Potomac River clays at Washington, D. C. The more important controlling conditions in the situations are: the identification, character, and stratification of the principal foundation soils contributing settlements: the compressibility and time-consolidation characteristics of the soils, the state of consolidation, and the natural initial stress conditions; type of structure, character of foundation loading, and permissible total and differential settlements and the possibility of improving foundation conditions by suitable methods and by fixing in advance the safe physical limits and time sequences and methods for construction.

The time-consolidation phenomenon was given major consideration in this paper. In view of the importance of the state of consolidation and of the natural initial stress in determining the useful region of the consolidation stress-strain curves to be used in both settlement and time-consolidation analyses, an essential test condition is the determination of the natural initial stress. Also for these compressible types of clay and organic soils, the stress increments should be made sufficiently small at all stages of the consolidation test, so as to minimize disturbance effects to the soil structure which affect both the compressibility and the time-delay characteristics for all subsequent stress increments and tend to an underestimation of the ultimate settlements, and an overestimation of the time-delay. In the case where one is dealing with overconsolidated clay-

soils, the settlements are overestimated and the time-delay is also overestimated. It would have been of interest to plot total expected settlements on each of the curves for comparison with the observed settlements to date.

MR. YASUMARU ISHII.²—The problem of evaluating secondary consolidation came up in a number of papers presented at this symposium and in some discussions from the floor. This discussion is concerned with a new approach in the interpretation of the nature of consolidation and in the theoretical treatment leading to solutions of the problem, which the writer developed in connection with his studies of the rate of subsidence phenomena at Osaka, Japan, from 1946 to 1950, caused by the lowering of ground-water levels. It is believed that this treatment will be of general interest to those who have had occasion to deal with these phenomena. This treatment attempts to clarify the nature of the phenomena and the concepts of secondary consolidation, and provides a working hypothesis, which goes somewhat beyond the work of Merchant and Taylor³ in 1940 and Taylor⁴ in 1942.

The usual practice in settlement studies has been to estimate the time-rate of consolidation of clay layers by Terzaghi's consolidation theory,⁵ which deals with primary consolidation only. Experience, however, has shown that secondary consolidation can reach considerable magnitude in the case of certain plastic clays and organic silts with a considerable time delay. Although the empirical procedure for determining secondary consolidation has been established by several authors, the problem of how secondary

² Yasumaru Ishii, Chief of Soil Laboratory, Research Section Port and Harbor Bureau, Ministry of Transportation, Japanese Government.

³ D. W. Taylor and W. Merchant, "A Theory of Clay Consolidation Accounting for Secondary Compressions," *Journal Mathematics and Physics* (1940).

⁴ D. W. Taylor, "Research on Consolidation of Clays S.82," Massachusetts Institute of Technology (1942).

⁵ K. Terzaghi, "Erdbaumechnik" (1925).

consolidation should be treated for practical purposes has not yet been fully clarified.

Consolidation Strains in Soils:

It is generally conceivable that the consolidation strains in soils consist of quasi-elastic strains of soil grains structure and the deformation caused by slippage of soil grains accompanied by volume decrease. The former sustains external load while the latter does not,

of pore-water flow to the deformation of consolidating soils was neglected, the quasi-elastic strain *versus* time curve $\epsilon_1 \sim t$, the creep *versus* time curve $\epsilon_2 \sim t$, might become as illustrated in Fig. 1. The $\epsilon_2 \sim t$ curve would have different shapes depending on the intensity of external load and nature of soil.

Mathematical Expression of Creep:

As is well known, secondary consolidation is indicated on the laboratory time

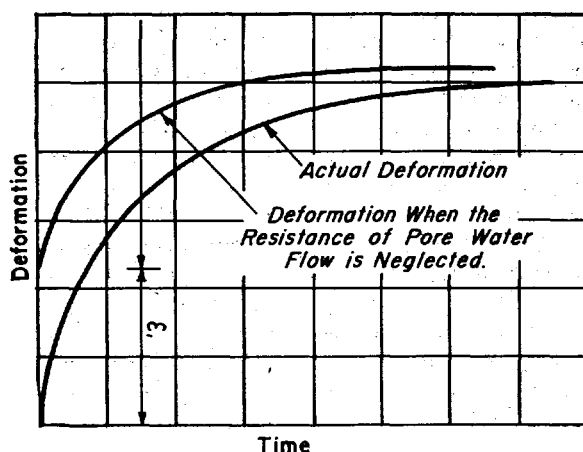


FIG. 1.—Quasi-Elastic Strain *versus* Time Curve.

a great part of strain energy being consumed in the interior of soil as internal work. But these two deformations are closely related to each other; for instance, the deformation by slippage of soil grains either increases or decreases the rigidity of elastic structure of soil grains.

The writer classified the consolidation strains of soils into these two parts, namely quasi-elastic strain, the time-lag of which might be caused mainly by pore-water flow, and creep (a gradual deformation caused by slippage of soil grains) the time lag of which is determined by behavior of the slippage as well as by pore-water flow. If the resistance

consolidation curve (deformation *versus* log time) by the straight line portion of the curve. On the basis of Kelvin's model⁶ for the representation of the behavior of visco-elastic materials, the following equation is proposed for the creep phenomena.

$$E_c = \sum_1^n \beta_n p (1 - e^{-\eta_n t}) \dots \dots \dots (1)$$

where:

E_c is the amount of creep,

p is the constant external load increment,

η_n is the time factor of creep,

⁶ A. Nadai, "Theory of Flow and Fracture of Solid," McGraw-Hill Book Co., Inc., New York, N. Y., p. 422 (1950).

β_n is the coefficient of creep, and t is the time.

By successively adopting a series of β_n and η_n , it is possible to express the creep portion of time-consolidation curve as exactly as desired by Eq 1. The idea of the summation of effects results in the essential difference between the writer's and Taylor's conclusion.

By changing the constant load increment p in Eq 1 to the variable, the following expression is obtained: (case $n = 1$)

$$E_c = \beta \int_0^t \left[-\frac{\partial}{\partial t} (1 - e^{-\eta(t-\tau)}) \right] p(\tau) d\tau \dots (2)$$

τ = parameter of time.

Then the load increment deformation relation becomes:

$$E = \alpha p$$

$$+ \beta \int_0^t \left[-\frac{\partial}{\partial \tau} (1 - e^{-\eta(t-\tau)}) \right] p(\tau) d\tau \dots (3)$$

α = coefficient of quasi-elastic strain.

The writer solved the case $n = 1$ at first, and the case $n = n$ was estimated by the results of the case $n = 1$.

Proposed Consolidation Equation and Solution:

Changing the term of $m_v \frac{\partial p}{\partial t}$ in Terzaghi's consolidation equation to the expression of Eq 3, the fundamental consolidation equation including the creep phenomena is derived as follows:

$$\alpha \frac{\partial p}{\partial t} + \beta \frac{\partial}{\partial t} \int_0^t \left[-\frac{\partial}{\partial \tau} (1 - e^{-\eta(t-\tau)}) \right] p(\tau) d\tau - k \frac{\partial^2 p}{\partial x^2} = 0 \dots (4)$$

This equation was readily solved by the operational method for the same boundary conditions as those that control in the consolidation test, yielding the final equation as follows:

$$u = p - \frac{4p}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n(\lambda_2 - \lambda_1)} [\lambda_2(\lambda_1 + \eta)e^{\lambda_1 t} - \lambda_1(\lambda_2 + \eta)e^{\lambda_2 t}] \sin \left(\frac{n\pi x}{2h} \right) \dots (5)$$

$$\bar{U} = \frac{\alpha}{\alpha + \beta} \left[1 - \frac{8}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} \frac{1}{(\lambda_2 - \lambda_1)\eta} \{ (\lambda_1 + \eta)\lambda_2 e^{\lambda_1 t} - (\lambda_2 + \eta)\lambda_1 e^{\lambda_2 t} \} \right] + \frac{\beta}{\alpha + \beta} \left[1 - \frac{8}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} \frac{1}{(\lambda_2 - \lambda_1)} (\lambda_2 e^{\lambda_1 t} - \lambda_1 e^{\lambda_2 t}) \right] \dots (6)$$

$$\left. \begin{matrix} \lambda_1 \\ \lambda_2 \end{matrix} \right\} = \frac{1}{2\alpha} \left[-(\alpha + \beta)\eta - k \left(\frac{n\pi}{2h} \right)^2 \right] \pm \sqrt{\left[(\alpha + \beta)\eta + k \left(\frac{n\pi}{2h} \right)^2 \right]^2 - 4\alpha k \eta \left(\frac{n\pi}{2h} \right)^2} \dots (7)$$

where:

u = hydrodynamical excess pore-water pressure,

$\bar{U}$ (per cent) = percentage consolidation, and

p = external load increment.

Examination of the Nature of Secondary Consolidation:

Although the solutions in the Eqs 5 and 6 appear to be very complicated, it is possible to evaluate the important relations between primary and secondary consolidation by the following numerical examinations:

$$\eta \gg \frac{k}{\alpha} \left(\frac{n\pi}{2h} \right)^2 \quad \left(X = \frac{k}{\alpha} \left(\frac{n\pi}{2h} \right)^2 / \eta < 0.3 \right)$$

$$u = p \left[1 - \frac{4}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} e^{-\frac{k}{\alpha + \beta} \left(\frac{n\pi}{2h} \right)^2 t} \right] \sin \left(\frac{n\pi x}{2h} \right) \dots (5')$$

$$\bar{U} = 1 - \frac{8}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} e^{-\frac{k}{\alpha+\beta} \left(\frac{n\pi}{2h}\right)^2 t} \dots (6')$$

These equations are exactly the same as those in Terzaghi's solutions except that the coefficient $\alpha + \beta$ appears instead of the coefficient m_v . This means, if the condition $X < 0.3$ were established, the process of consolidation is represented satisfactorily by Terzaghi's equation, that is, the portion of creep which has a large value of η in satisfying the condition $X < 0.3$ behaves in the consolidation process as if it were elastic strain. (Practically the time lag of this creep can be assumed to be zero.)

$$\eta \ll \frac{k}{\alpha} \left(\frac{n\pi}{2h}\right)^2 \quad \left(X = \frac{k}{\alpha} \left(\frac{n\pi}{2h}\right)^2 / \eta > 50 \right)$$

$$u = p \left[1 - \frac{4}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} e^{-\frac{k}{\alpha} \left(\frac{n\pi}{2h}\right)^2 t} \right] \sin \left(\frac{n\pi x}{2h} \right) \dots (5'')$$

$$\bar{U} = \frac{\alpha}{\alpha + \beta} \left[1 - \frac{8}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} e^{-\frac{k}{\alpha} \left(\frac{n\pi}{2h}\right)^2 t} \right] + \frac{\beta}{\alpha + \beta} [1 - e^{-\eta t}] \dots (6'')$$

The Eq 5'' and the first term of the Eq 6'' are exactly the same as those in Terzaghi's solutions. The second term of Eq 6'' represents the expression of pure creep. Moreover, the value of $\frac{k}{\alpha} \left(\frac{n\pi}{2h}\right)^2$ is far larger than the value of η . Therefore, Eq 6'' shows two separated curves: one is the curve of pure primary consolidation and the other the curve of pure creep. This means that the portion of creep which has small value of η providing the condition $X > 50$, represents itself as a pure creep in the consolidation process having no relation with primary consolidation.

From the above derivations the following conclusions are obtained:

1. Primary consolidation consists of quasi-elastic strain and the portion creep which has the large value of η ($X < 0.3$). Secondary consolidation is the portion of creep having small values of η ($X > 50$). The end point of primary consolidation can be determined by Eq 5, as the case of $u = 0$. Hydrodynamically, it means that external load increment is completely sustained by soil grains structure. The end point is easily obtained by the successive approximation method from Eq 5.

2. The magnitudes of primary and secondary consolidation change by the value of X , $\left(= \frac{k}{\alpha} \left(\frac{n\pi}{2h}\right)^2 / \eta \right)$. The value X can also change by the magnitude of the thickness of layer, h . Therefore, however alike all the other conditions might be, the coefficients of volume change, which are computed from the primary consolidations, can have different values depending on the thickness of clay layers.

3. The thickness of consolidating clay layers in the field may reach in general 20 m or more, and, on the other hand, the thickness of sample at laboratory is usually 2 ~ 5 cm. This tremendous difference between the thickness of layers has the important meaning.

If, for example, $k = 10^{-6}$ cm per min, $\alpha = 10^{-4}$ cm per g and $n = 1$,

$$(a) \quad 2h = 2\text{cm.} \quad X = 50$$

$$\frac{k}{\alpha} \left(\frac{n\pi}{2h}\right)^2 = 2.5 \times 10^{-2} \text{ 1/min}$$

$$\text{then, } \eta = 5 \times 10^{-4} \text{ 1/min} = 0.72 \text{ 1/day}$$

This value of η is a usual one and creep begins to appear about one-half day after application of load and it continues about three or four days. The reason why secondary consolidation appears

even in the case of a stiff clay in laboratory test, is explained by these derivations.

$$(b) \quad 2h = 30m, \quad X = 0.3$$

$$\frac{k}{\alpha} \left(\frac{n\pi}{2h} \right)^2 = 10^{-8} \text{ l/min}$$

$$\text{then } \eta = 3 \times 10^{-8} \text{ l/min} = 1.58 \times 10^{-2} \text{ l/year}$$

The portion of creep which has the time lag of 0.0158 per year might have negligible magnitude, and, therefore, almost all portions of creep will be included into primary consolidation. From this standpoint the following conclusion is quite reasonably derived: Terzaghi's consolidation theory can be applied without any corrections for practical consolidation problems in the field, except special case (settlement by gradual shear deformation, settlement of peaty silt layer, and so on), despite its simple assumption on the behavior of the deformation of the consolidating clay layers. However, it should be noted in this case that the coefficient $\alpha + \Sigma_1^n \beta_n$ must be used as the coefficient of volume change.

The value computed from primary consolidation should not be used. It has no meaning by itself.

Practical Suggestions for the Analysis of Secondary Consolidation:

In order to evaluate the rate of settlement of clay layers, the magnitude of the coefficient of permeability, volume change, $(\alpha + \Sigma_1^n \beta_n)$, and the thickness of layer must be estimated. The coefficient of permeability is determined either by direct measuring method or by consolidation test. For determination of the coefficient of permeability by consolidation test, it is necessary to pick up the pure primary consolidation from time-settlement curve.

The writer suggests the following ap-

proximate method for separation procedure of primary and secondary consolidation:

Two scales serve for this purpose, one is the type of

$$m_v \left[1 - \frac{8}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} e^{-[(k/m_v)(n\pi/2h)^2 t]} \right]$$

and the other the type of $\beta[1 - e^{-\eta t}]$, having the different magnitudes of m_v and β . (Ten curves for one scale are enough.)

The difference between the magnitude of $\frac{k}{m_v} \left(\frac{n\pi}{2h} \right)^2$ and η results only in parallel displacement of curves to time axis (log of t). Therefore, by shifting these scales parallel to time axis, and by finding two curves, the combination of which corresponds with the observed curve, the approximate value of primary consolidation can be obtained. The successive approximation method is very useful for this purpose.

The coefficient of volume change is very difficult to determine. Soil engineers have estimated this coefficient from either primary consolidation or 24 hr consolidation. It is clearly a mistake. Total consolidation must be used. The only method for determination of total consolidation is a long duration loading test.

By adoption of total consolidation, the estimated rate of settlement becomes much smaller, despite the larger final settlement.

Discussion of the Assumption Derived by the Writer:

The process of primary consolidation is hydrodynamically the process of transmission of load, which was sustained temporarily by the resistance of pore-water flow, to soil grains structure. The end of primary consolidation means that the total external load increment is

sustained by strain of soil. Secondary consolidation does not participate.

The writer introduced quasi-elastic strain purely theoretically as the strain which resists to external load. It could not have practical correspondence to the features of void-pressure relation at the present stage of soil mechanics. However, the existence of primary consolidation is an evidence of the existence of load-sustained strain of soil. So far as the problem of time lag of soil consolidation is concerned, the existence of load sustainable strain is a necessary and suffi-

cient condition for the purposes of analysis.

The properties of soil as a material, elastic or plastic, reversible or irreversible, creep and elastic properties and so on, need further extensive studies.

Acknowledgment:

The writer wishes to acknowledge the advice of G. P. Tschebotarioff of Princeton University and D. M. Burmister of Columbia University in the presentation of these concepts.

APPENDIX

The fundamental equation is written:

$$\alpha \frac{\partial p}{\partial t} + \beta \frac{\partial}{\partial t} \int_0^t \left[-\frac{\partial}{\partial \tau} (1 - e^{-\eta(t-\tau)}) \right] p(\tau) d\tau - k \frac{\partial^2 p}{\partial x^2} = 0 \dots (4)$$

By Laplace's transformation formula, the second term of Eq 4 is transformed as follows: ($p \rightarrow u'$)

$$\frac{\beta \eta p}{p + \eta} u' \quad \left(p \equiv \frac{\partial}{\partial t} \right)$$

Eq 4 becomes:

$$\alpha p u' - \frac{\beta \eta p}{p + \eta} u' - k \frac{\partial^2 u'}{\partial x^2} = 0 \dots (8)$$

The initial condition $t = 0, u' = 0$

The boundary condition $x = 0 \left. \begin{array}{l} u' = p \\ = 2h \end{array} \right\} u' = p$

Changing $u' \rightarrow p - u$, the above conditions become:

$$\left. \begin{array}{l} t = 0 \quad u = p \\ x = 0 \\ \quad \quad 2h \end{array} \right\} u = 0$$

As a function of the solution, the expression

$$u = \sum A_n(t) \sin \left(\frac{n\pi x}{2h} \right) \dots (9)$$

is adopted, which satisfies the boundary condition. Then

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial}{\partial t} \sum A_n(t) \sin \left(\frac{n\pi x}{2h} \right) \\ \frac{\partial^2 u}{\partial x^2} &= - \sum \left(\frac{n\pi}{2h} \right)^2 A_n(t) \sin \left(\frac{n\pi x}{2h} \right) \end{aligned}$$

following.

By substituting these expressions in Eq 8, the following equation is obtained:

$$\alpha p A_n(t) + \frac{p\eta\beta}{p + \eta} A_n(t) + k \left(\frac{n\pi}{2h} \right)^2 A_n(t) = 0 \dots (10)$$

This equation is a linear equation of second order, and, therefore, the following expression can be used as a solution:

$$A_n(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \dots (11)$$

C_1, C_2 : dimensionless quantities,
 λ_1, λ_2 : time factors

From Eqs 11 and 10, λ_1 and λ_2 are written as follows by comparing coefficients of each term.

$$\left. \begin{array}{l} \lambda_1 \\ \lambda_2 \end{array} \right\} = \frac{1}{2\alpha} \left[-(\alpha + \beta)\eta - k \left(\frac{n\pi}{2h} \right)^2 \pm \sqrt{\left[(\alpha + \beta)\eta + k \left(\frac{n\pi}{2h} \right)^2 \right]^2 - 4\alpha k \eta \left(\frac{n\pi}{2h} \right)^2} \right] \dots (7)$$

Then Eq 9 becomes:

$$u = \sum_{n=1}^{\infty} (C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}) \sin \left(\frac{n\pi x}{2h} \right) \dots (12)$$

On the other hand, p is expressed by Fourier's series:

$$p = \sum_{n=1,3,5,\dots}^{\infty} \frac{4p}{n\pi} \sin \left(\frac{n\pi x}{2h} \right) \dots (13)$$

From Eqs 8, 12 and 13 the following relations are obtained by using initial condition:

$$\left. \begin{aligned} C_1 + C_2 &= \frac{4p}{n\pi} & n: \text{odd} \\ &= 0 & n: \text{even} \end{aligned} \right\} \dots (14)$$

On the other hand, at $t = 0$, creep = 0, therefore, computing $\frac{p_n \beta}{p + n} A_n(t)$, and equalizing

to zero:

$$\frac{4p\beta}{n\pi} - \beta\eta \left[\frac{C_1}{\lambda_1 + \eta} e^{\lambda_1 t} + \frac{C_2}{\lambda_2 + \eta} e^{\lambda_2 t} \right] = 0$$

$t = 0$

$$\frac{4p}{\eta\pi} = \eta \left[\frac{C_1}{\lambda_1 + \eta} + \frac{C_2}{\lambda_2 + \eta} \right] \dots (15)$$

From Eqs 14 and 15, C_1 and C_2 are obtained

$$\left. \begin{aligned} C_1 &= -\frac{\lambda_2(\lambda_1 + \eta)}{(\lambda_1 - \lambda_2)\eta} \cdot \frac{4p}{n\pi} \\ C_2 &= \frac{\lambda_1(\lambda_2 + \eta)}{(\lambda_1 - \lambda_2)\eta} \cdot \frac{4p}{n\pi} \end{aligned} \right\} \dots (16)$$

By these values Eqs 5 and 6 are easily introduced.