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DISCUSSION

B. LADANYI[1]—In their two papers, the one published recently in Geotechnique,[2] and the one submitted to this symposium, the authors have given a very interesting approach to the problem of defining the general stress-strain behavior of a normally consolidated clay within a given region of the water content.

In principle their method consists in covering the stress-space defined by the isotropic and the deviatoric components of the stress tensor by two systems of stress paths which intersect each other, and in finding at each intersection point the values of the volumetric and the deviatoric components of the strain tensor.

The strain path corresponding to a given stress path within the defined region is then found by assuming that the law of superposition of strains is valid, provided that the strain components are not changing sign during the process. The stress paths in the stress space chosen by the authors for defining the general stress-strain behavior of a normally consolidated clay correspond to a consolidation process (defined by drained triaxial compression tests in which p is increasing and σ_1'/σ_3' is held constant) and to a constant-volume process (defined by undrained triaxial compression tests at different water contents).

The method proposed by the authors is very useful and well adapted to the problem of a normally consolidated clay.

It may, however, be interesting to mention that the stress-strain behavior of a soil within a given region of density can be defined as well by the types of tests different from those chosen by the authors. In fact a method, very similar in principle to that presented by the authors was proposed fo sand by the writer some years ago.[3]

In this latter method the stress-strain behavior of sand within a given region of density was defined by the following two types of tests: an isotropic consolidation test, and a series of drained triaxial compression tests in which the value of the mean effective normal stress $\sigma_m' = p$ was held constant, each of them beginning at a given point on the consolidation curve. In such a way the stress space for a triaxial test was covered by an orthogonal system defined by the space diagonal ($\sigma_1' = \sigma_2' = \sigma_3'$) as the abscissa and the lines σ_m' = const. as ordinates (Fig. 8). At each point of the system the values of the volumetric and the deviatoric strain components were known. However, for the purpose of performing a simple graphical interpolation, it was found more convenient to plot the results of the series of tests in a combined space stress-strain

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[2] K. H. Roscoe and H. B. Poorooshasb, "A Theoretical and Experimental Study of Strains in Triaxial Compression Tests on Normally Consolidated Clays," *Geotechnique*, Vol. XIII, No. 1, 1963, pp. 12–38.

[3] B. Ladanyi, "Etude des Relations Entre les Contraintes et les Déformations Lors du Cisaillement des Sols Pulvérulents," *Annales des Travaux publics de Belgique*, No. 3, 1960, pp. 241–274.

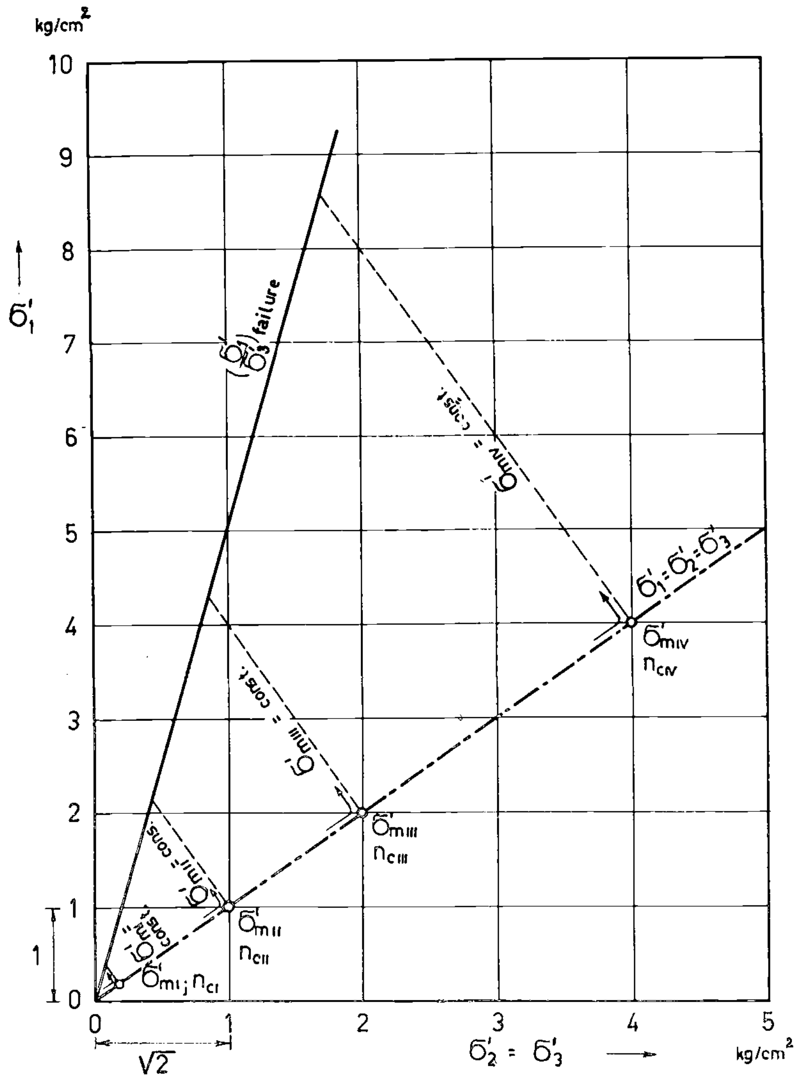

FIG. 8—Loading Paths for Drained, Effective Mean Normal Stress Constant Tests.

diagram, as that shown in Fig. 9. The upper part of the figure gives the relationship between the shear strain γ and the values of σ_1'/σ_3' and σ_m', while the lower part relates the shear strain γ to the total volumetric strain e produced by compression and dilatancy. In this figure, n_c denotes the porosity of the specimen at the beginning of the shear test.

By using this figure it is possible by a simple interpolation to find the stress-strain curves for the sand either if a stress path defined by the relationship between σ_m' and σ_1'/σ_3' is given, or if the strain conditions are imposed as in a constant-volume test ($e = 0$) or in an oedometer test ($e = \gamma$).

The so defined stress-strain relationship for the sand has been used as a basis for solving the problem of the expansion of cavities in sand.[4]

It may be mentioned that the method originally used for sand can easily be adapted for the application in the case of a saturated clay.

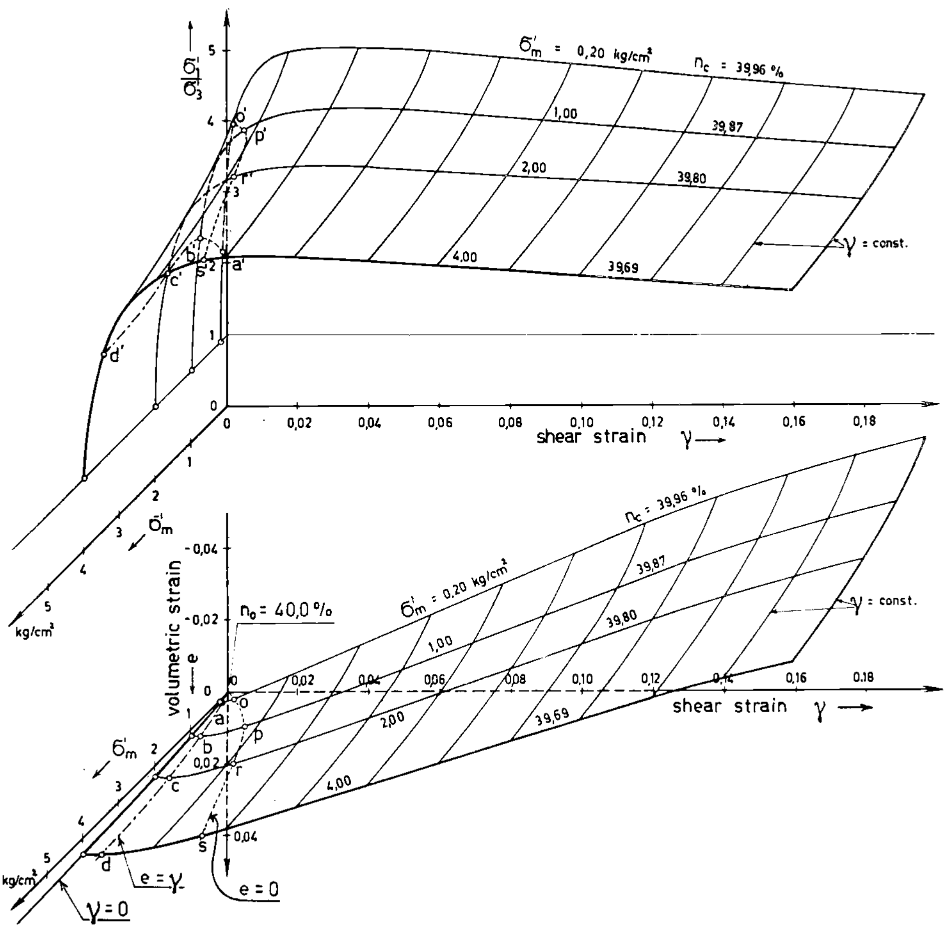

FIG. 9—Stress-Strain Surfaces for a Dense Sand.

ROBERT L. KONDNER[5]—The research reported by Poorooshasb and Roscoe on graphical techniques, as well as the analytical work previously reported by Roscoe and Poorooshasb, for the strain response of normally consolidated clays tested in triaxial compression under an imposed stress path is extremely interesting and merits careful consideration by soil mechanicians. However, in attempting to evaluate the usefulness of the technique to represent the Kaolin re-

[4] B. Ladanyi, "Etude Théorique et Expérimentale de L'expansion dans un Sol Pulvérulent d'une Cavité Présentant une Symétrie Sphérique ou Cylindrique," *Annales des travaux publics de Belgique*, Nos. 2–4, 1961, pp. 365–406.

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sponse as indicated in the example presented in Fig. 7, the writer has experienced difficulty in duplicating the results given in Fig. 7 by Poorooshasb and Roscoe.

According to the authors' Eq 3, the strain increment due to a change of stress, is written

$$\delta\epsilon' = \delta\epsilon'_v + \delta\epsilon'_\eta \qquad (3)$$

where $\delta\epsilon'_v$ can be obtained directly from Fig. 3 as a function of η. The magnitude of $\delta\epsilon'_\eta$ can be obtained from Eq 4, as

$$\delta\epsilon'_\eta = \left(\frac{d\epsilon'}{dv'}\right)_\eta \cdot \delta v' \qquad (4)$$

where $(d\epsilon'/dv')_\eta$ may be obtained from Fig. 5. The increment of volume change, $\delta v'$, is given by Eq 5 as

$$\delta v' = -\frac{1}{3}\left(\frac{\delta w}{w + \frac{1}{G_s}}\right) \qquad (5)$$

in which the magnitude of δw is found by drawing the stress path CD of Fig. 7 on to the water content contours of Fig. 1. By such a procedure Poorooshasb and Roscoe indicate that one may obtain the theoretical points of Fig. 7. The writer has tried such a procedure and has obtained values of ϵ' which are substantially higher than those indicated on Fig. 7. As an example, consider the case of $\eta = 0.5$. For $\eta = 0.5$, Fig. 3 gives $\delta\epsilon'_v$ equal to 3 per cent, and Fig. 5 gives $(d\epsilon'/dv')_\eta$ equal to 3.09. The superposition of the stress path CD of Fig. 7 directly on to Fig. 1 is not very convenient because a large portion of the stress path lies outside of the region of moisture content contours. However, use of the similarity assumption given by Roscoe and Poorooshasb allows one to draw a similar stress path at a different location within the region of moisture content contours of Fig. 1. The writer has tried this but has found that the values of δw seem to differ, depending upon the location of the similar stress path in Fig. 1.

In addition to the above discrepancy, the values of w and G_s needed in Eq 5 were not given by Poorooshasb and Roscoe. Since the strain process is assumed to be one of superposition of effects, the writer has used a value of $\delta v'$ equal to 1.13 per cent given in Fig. 7 by Poorooshasb and Roscoe for $\eta = 0.5$. Substitution into Eq 4 gives

$$\delta\epsilon'_\eta = (3.09)(1.13) \times 10^{-2} = 3.49 \text{ per cent} \qquad (6)$$

Thus, the total strain increment given by Eq 3 is

$$\delta\epsilon' = 3.00 + 3.49 = 6.49 \text{ per cent} \qquad (7)$$

which is considerably higher than the value of ϵ' given by Poorooshasb and Roscoe in Fig. 7 for $\eta = 0.5$.

It might be beneficial to others who are interested in application of the work of Poorooshasb and Roscoe, and certainly very helpful to the writer, if the authors would present the following information.

1. Give the values of w_a, λ, and the function $F(q/p)$ of Eq 1 as well as w_h for the Kaolin tested and reported in Fig. 7.

2. Show the correct stress path CD of Fig. 7 superimposed on Fig. 1, and indicate the location of the points corresponding to the theoretical points given in Fig. 7.

3. Give the values of w and G_s associated with the Kaolin of Fig. 7 and needed for use in Eq 5.

4. Give a table listing the values of $\delta\epsilon'_v$, $(d\epsilon'/dv')_\epsilon$, δw, $\delta v'$, $\delta\epsilon'_\eta$, and $\delta\epsilon'$ for values of η of 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.75, and 0.8 associated with the drained test of Fig. 7.

H. B. POOROOSHASB AND K. H. ROSCOE (*authors' closure*)—Dr. Kondner has noticed that the authors of the paper have failed to report certain data which in his views are of importance. However, it must be stated that the authors were basically concerned with the presentation of the fundamental concept rather than its application to some particular situations. Indeed in the short space allocated to them it was not possible to include all the information required by Dr. Kondner, and the authors felt that the interested reader would use their previous publication, in which all the necessary and relevant data are provided.

Concerning the difficulty that Dr. Kondner is experiencing in evaluating strains and the lack of correlations he obtains between his theoretical and experimental points, it appears to the authors that he has failed to consider the salient point in the paper. The principle of superposition suggested in the paper must be used incrementally. The smaller the increments of stress path used, the more accurate will be the results obtained. The stress increment used by Dr. Kondner in his discussion is very large and hence not acceptable for this situation. In their previous publication[2] the authors devoted a good deal of space to explain the dependence of strain on stress path as well as the validity of the principle of superposition.[6]

Dr. Kondner states that for $\eta = 0.5$, $\delta v' = 1.13 \times 10^{-2}$. He uses this value to obtain $\delta\epsilon'_\epsilon$. This procedure is erroneous since $v' = 1.13 \times 10^{-2}$ and not $\delta v'$.

For the stress increment between $\eta = 0.4$ and $\eta = 0.5$, $\delta v' = 0.37 \times 10^{-2}$ (calculated from Fig. 1) and hence $\delta\epsilon'_\epsilon = 0.37 \times 3.09 = 1.15$ per cent. $(3.09 = (d\epsilon'/dv')_\epsilon = 0.5)$. Furthermore, from Fig. 3 of the paper $\delta\epsilon'_v = 1.1$ per cent. Thus

$$\delta\epsilon'_{\eta=0.4 \text{ to } \eta=0.5} = \delta\epsilon'_v + \delta\epsilon'_\eta$$
$$= 1.15 + 1.1 = 2.25 \text{ per cent}$$

This added to the value of strain at $\eta = 0.4$, that is, $(\epsilon')_{\eta=0 \text{ to } \eta=0.4} = 2.4$ per cent gives the total strain from beginning of test which is

$$\epsilon'_{\eta=0 \text{ to } \eta=0.5} = 2.4 + 2.25 = 4.65 \text{ per cent}$$

and is in good agreement with observed experimental value of 4.4 per cent.

[6] See, in particular, the last three lines on p. 28 of the authors' previous publication.