GENERAL DISCUSSION

MR. J. T. RANSOM[1] (*presented in written form*).—Messrs. Freudenthal and Weibull have recently demonstrated and discussed the statistical nature of the life to failure in fatigue. A brief discussion of the statistical aspects of the endurance limit might be in order.

To illustrate how one may be deceived in the determination of an endurance limit when using limited data, I should like to show a few examples. Shown in Fig. 1 is an *S-N* curve determined by ten specimens. Two specimens have failed at 47,000 psi while two specimens at 46,000 psi have run unbroken to ten million cycles. Clearly, the endurance limit falls between 46,000 and 47,000 psi. Another *S-N* curve determined with ten specimens is shown in Fig. 2. From the data plotted there is little doubt that the endurance limit is near 40,000 psi. I can show eight more such curves in each of which the endurance limit is fixed without question, and a fracture curve can be drawn agreeing closely with the data. I shall show only a summary of the curves without the individual points, Fig. 3. The endurance limits range from below 40,000 psi to above 48,000 psi, a variation of perhaps 20 per cent. The crux of this demonstration is that these data were obtained from specimens all of which came from the same bar of steel. The point to be made is that each *S-N* curve and endurance limit appears to be determined without question by the ten specimens, and yet the endurance limits differ by 20 per cent.

Obviously the endurance limit is a statistical property and must be determined by statistical procedures. The problem is slightly more difficult than in the determination of life to failure at constant stress. In the latter case the lives of a number of specimens can be accurately measured and common techniques used to find the mean and standard deviation. But the endurance limit of each specimen cannot be accurately measured. If a specimen is tested at a given stress and fails, the only information gained about its endurance limit is that the given stress was above the endurance limit. Conversely, a specimen which runs out ten million cycles tells only that the test stress is below its endurance limit. In neither case is the specimen suitable for a second approximation.

The problem is similar to that of determining the lethal dose of poison on a group of insects or the critical energy of blow required to detonate an explosive. There is a simple statistical procedure for these cases and this procedure can be used for the estimation of the mean endurance limit and the standard deviation. It is called "staircase testing."[2] It is best illustrated by example, Fig. 4. One generally has an approximate idea of the average endurance limit. The first specimen is run at this

[1] Materials of Construction Section, Engineering Research Lab., E. I. du Pont de Nemours & Co., Wilmington, Del.

[2] W. J. Dixon, and A. M. Mood, *Journal*, Am. Statistical Assoc., Vol. 43, pp. 109–126 (1948).

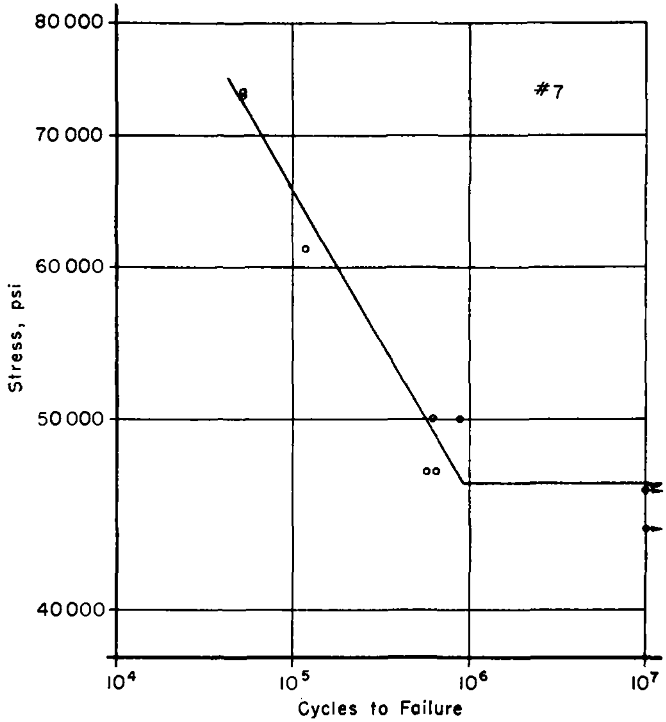

FIG. 1.—An *S-N* Curve with Ten Specimens.

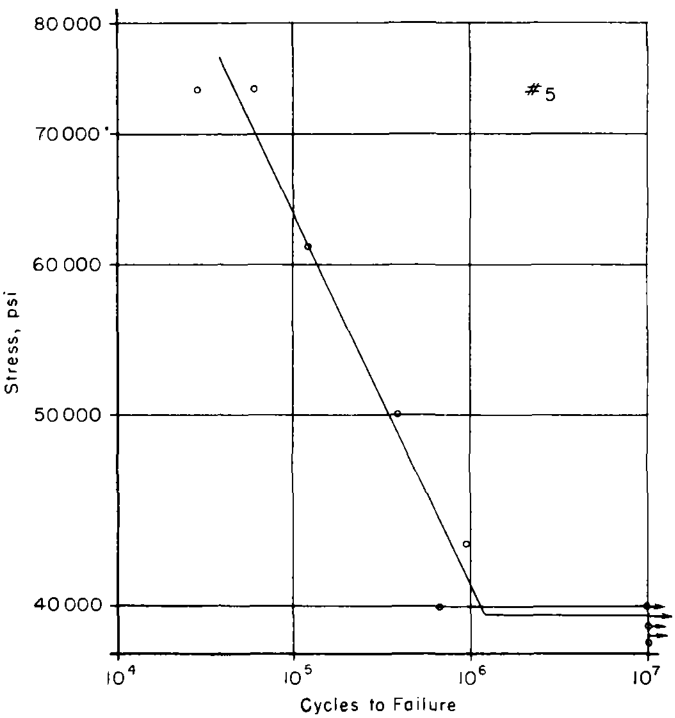

FIG. 2.—A Second *S-N* Curve with Ten Specimens.

stress. In this example we started at 46,500 psi. The first specimen failed as indicated by the "X", so the second was tested at one stress step lower, or by the "0." Then the fourth was tested one step higher, etc. You can see that each specimen is tested at a stress one step above or below the test stress of

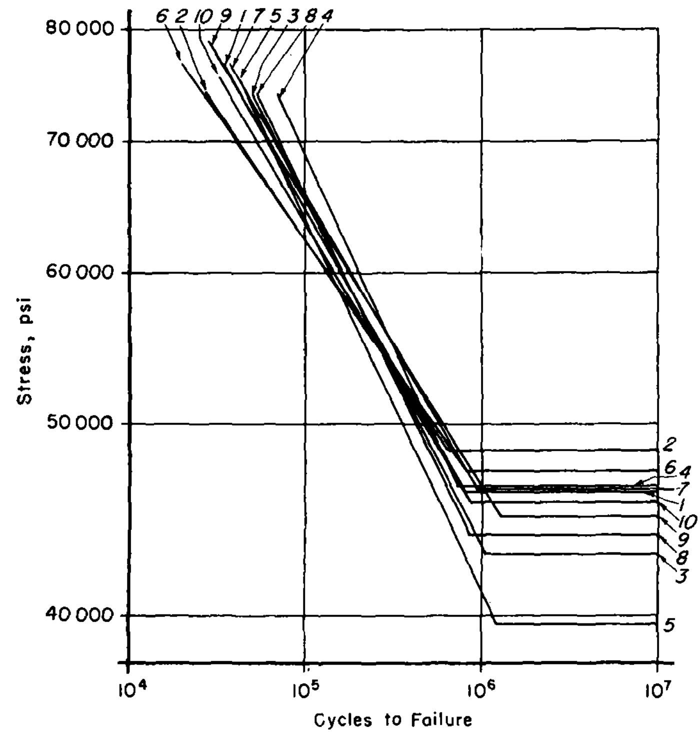

FIG. 3.—Summary of *S-N* Curves, Each with Ten Specimens, from Same Bar of Steel (Specimens Transverse to Forging Fiber).

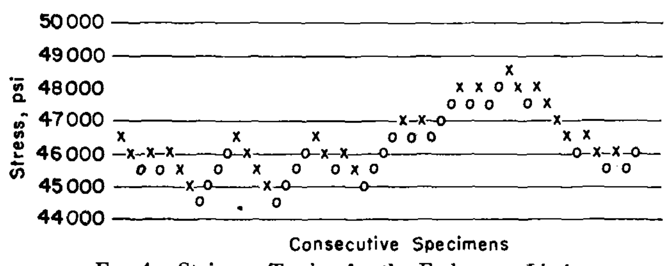

FIG. 4.—Staircase Testing for the Endurance Limit.

46,000 psi. The step size was chosen as 500 psi, although it was later found that 2000 psi is a better value for most purposes. The second specimen also failed, so that the third was tested at 45,500 psi. The third ran out as indicated the previous specimen, depending upon whether the previous specimen ran out or failed, respectively. In the present example, 54 specimens were tested.

The use of these data for computation of the mean or average endurance limit and the standard deviation is extremely simple if more than 25 specimens have been tested. The procedures are now readily available in the literature. Here the mean endurance limit, defined as the stress at which one can expect 50 per cent failures, is 46,270 psi with a standard error of 500 psi. The standard deviation, a measure of variability, is 2900 psi with a standard error of 1600 psi. With these results we can now say, just for example, that the most probable stress which will produce only $2\frac{1}{2}$ per cent failures is 40,470 psi, and the most probable stress which will result in $97\frac{1}{2}$ per cent failures is 52,070 psi. We can also calculate confidence intervals for these various estimates.

We are now in a position to compare the endurance limits of different materials, either as to averages or dispersion; and we can state a degree of confidence in our conclusions, For example, we can state that we are 95 per cent confident that the differences observed represent real phenomena and are not due to scatter and sampling errors. Only in this way can fatigue research be conducted on a sound scientific basis.

Numerous discussors at these meetings have declaimed against the running of laboratory fatigue tests. I feel that someone must defend this practice.

As I see it, there are two separate problems. The first is to be able to design using the present knowledge of fatigue, and the second is to determine the fundamental cause of fatigue failure.

With regard to the first problem, we have to admit that the present state of our knowledge is so poor that design data can be obtained only by running full size tests. But I hope that the state of our knowledge will improve so that it is not always necessary to run full scale tests. I should like to see more work directed at finding out why laboratory tests cannot be correlated with full scale tests, and the best way to approach this problem is to study the fundamental effects of each of the individual factors which could be involved in such a correlation.

Now I don't see how we will ever find out much of a fundamental nature about fatigue by running full scale tests. There are too many variables involved to be able to sort out their separate effects. This fundamental study is the job of the laboratory scale test.

I believe that both types of fatigue research should be promoted. Full scale tests are necessary at this time because it is the only way to obtain reliable design information. But laboratory scale tests are just as important if we are ever to make real progress.

MR. E. EPREMIAN[3] (*presented in written form*).—The purpose of this discussion is to describe briefly without details or interpretation some research which has been done on the statistical behavior of the fatigue of metals. The complete paper on this work will be submitted to the Society for publication at a later date.[4] The statistical nature of fatigue properties is now well recognized, but the influence of metallurgical factors on this behavior has not been known. While research has shown that the fatigue life and endurance limit vary statistically, the dependence of this variability on such factors as composition, microstructure, hardness, and inclusion rating has not been evaluated. It was toward this objective that considerable experimental work was directed.

A variety of ferrous materials including annealed Armco iron, a plain carbon steel, and an alloy steel heat treated to different microstructures were tested in

[3] Research Associate, Metals Research Lab., Carnegie Institute of Technology, Pittsburgh, Pa.

[4] See program for 1952 Annual Meeting, Am. Soc. Testing Mats.

fatigue to obtain the statistics of the fracture curve and endurance limit. For the most part, this work was done with the pneumatic vibrating cantilever fatigue test machine which is capable of detecting crack initiation, thus providing a better criterion of fatigue failure than the usual R. R. Moore test which measures cycles to complete fracture. Analysis and comparison of the experimental results by statistical methods indicated the influence of the metallurgical variables on the statistical behavior.

In brief, it was discovered that the most important single factor in producing variability is the inclusion rating, and that other factors such as composition, microstructure, etc., are of secondary importance. It should, of course, be understood that, other things being equal, these secondary factors can have an effect. For example, with a given composition and inclusion rating, the statistical variation in the endurance limit was greater for a quenched-and-tempered structure than that obtained with a quenched-and-spheroidized structure. On the whole, however, inclusions play a dominant role in this behavior. Further it was found by analysis of data reported in the literature for nonferrous metals that these materials have less statistical variation in fatigue life than that obtained with ferrous materials, presumably because the former materials have fewer inclusions and inhomogeneities.

In addition, a number of other aspects of the problem were investigated which can only be mentioned here:

A detailed study was made of the dependence of the statistical variations in fatigue life on stress levels in the fracture range and a theoretical interpretation of the observed behavior has been developed.

The form of the complete S-N diagram and the methods of plotting it have been analyzed and a new method of presenting the data whereby a straight-line plot is obtained has been proposed.

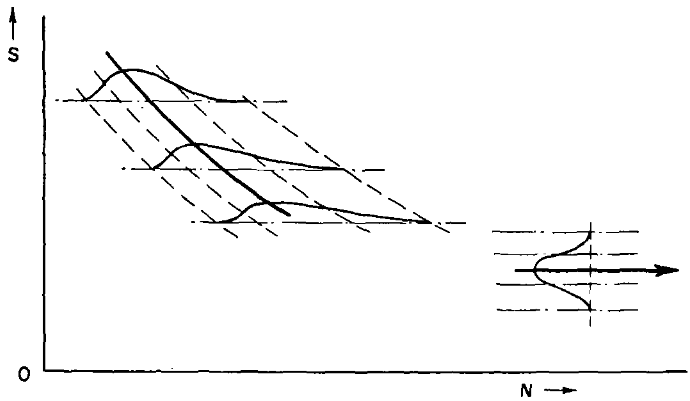

FIG. 5.—Statistical Features of S-N Data.

The location of crack initiation in R. R. Moore fatigue specimens has been analyzed and shown to behave in a statistical manner.

The understressing effect has been investigated and it may be interpreted, in part if not wholly, as a statistical phenomenon based purely on selectivity.

CHAIRMAN R. E. PETERSON[5] (*by letter*).—The report by Mr. Ransom deals with a method applicable to the "en-

[5] Manager, Mechanics Div., Westinghouse Research Labs., Westinghouse Electric Corp., East Pittsburgh, Pa.

durance limit" portion of an S-N diagram. Not all materials show such a limit, but an important number of materials do possess this property. In such a case a practical question facing the experimenter is how to obtain the most useful information from a given number of test specimens. The answer depends on the intended use of the results.

If for example the purpose is to obtain a "fatigue notch factor", K_f, for steel specimens for comparison with a theoretical stress concentration factor, K_t, then perhaps the best use of the specimens is to test all in the limiting region, employing the "staircase method" described by Ransom. Similarly, if one is interested in comparing strength theories for fatigue of steel specimens, as was recently done by Majors, *et al.*[6], it would again seem wise to test all specimens in the limiting condition since this is all that is finally used. Points high up on the diagram have no value in this particular case.

On the other hand, the intended use could refer only to finite life, as is the case for a number of practical problems. In this case the approach would be along the lines described in the paper by Freudenthal.

Then there is the third possibility—the intended use is not known specifically. This is the position of a material supplier. In this case one must decide what portion of the specimens to devote to the finite and to the limiting regions (see Fig. 5).

One of our tasks for the future will will be to work out recommendations for the planning of fatigue investigations, in terms of objectives of the investigation, number of specimens, and confidence limits.

[6] H. Majors, B. D. Mills, and C. W. MacGregor, "Fatigue under Combined Pulsating Stresses," *Transactions*, Am. Soc. Mechanical Engrs., Vol. 71, p. 269 (1949).