DISCUSSION

J. H. Underwood[1] (*written discussion*)—The author should be complemented for considering the bending constraint of a finite specimen as a separate and important effect on the K_I calibration. Often the back surface of finite specimens is considered to affect K_I in the same basic manner as does the front surface. However, whereas the presence of the front surface produces no basic change in loading conditions and causes in the order of a 10 percent change in K_I, the presence of the back surface involves a change from a semi-infinite to a finite geometry. The accompanying change from infinite to finite resistance to gross specimen bending often becomes the dominant factor in determining K_I for edge-notched finite geometries.

Two aspects of Buchalet's work can be compared with some recent work in the literature and discussed in relation to bending constraint effects. They are (1) his K_I expressions for an internal, circumferencial notch in hollow cylinders under axial tension; and (2) his representation of a long, shallow surface flaw by using a continuous flaw.

Swedlow and Ritter[2] have considered circumferentially notched cylinders from a different point of view, that is, crack front curvature effects. Their results can nevertheless be compared with those under discussion. The form of Buchalet's K_I expressions (Eqs 3 and 4) is a good basis of comparison.

$$K_I = 1.12 \sqrt{\pi}\, \sigma \sqrt{a} \cdot F \tag{9}$$

His K_I expressions reduce to the form of Eq 9 for the situation of a uniform axial stress, σ, applied to a cylinder with an internal, circumferential notch of depth, a. His correction factor, F, is a function of the notch-depth to wall-thickness ratio, a/t. Buchalet presents correction factors for two loading conditions (see Figs. 3 and 4): a flat plate where the movement of the back wall is "totally prevented," which can also apply to a cylinder with a large amount of bending constraint; and a large cylinder with a small amount of bending constraint due to a large radius to wall-thickness ratio, r/t. These correction factors are listed in Table 3 along with factors of the same form from Swedlow and Ritter's work and from the Gross et al[3] analysis of a single-edge-notched plate.

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[2] Swedlow, J. L. and Ritter, M. H. in *Stress Analysis and Growth of Cracks, Part I, ASTM STP 513*, American Society for Testing and Materials, 1972, pp. 79–89.

[3] Gross, B., Srawley, J. E., and Brown, W. F., "Stress Intensity Factors for a Single-Edge-Notch Tension Specimen by Boundary Collocation of a Stress Function," Technical Note D-2395, NASA, Aug. 1964.

TABLE 3—*Comparison of notched cylinder stress intensity factors* $F = K/1.12\,\sigma\sqrt{\pi a}$.

Reference Geometry	Buchalet: Constrained Plate F_1	Buchalet: Large Cylinder $F_1^{(C)}$	Swedlow and Ritter: Small Cylinder F_2, $r/t = 2$	Swedlow and Ritter: Large Cylinder F_8, $r/t = 8$	Gross et al: SEN Plate F_G
$a/t = 0$	1.00	1.00	1.00	1.00	1.00
0.1	1.02	1.03	1.01	1.04	1.06
0.2	1.05	1.14	1.02	1.16	1.22
0.3	1.09	1.33	1.06	1.32	1.48
0.4	1.15	1.56	1.11	1.58	1.88
0.5	1.22	1.82	1.20	1.88	2.51

Swedlow and Ritter's small cylinder results for $r/t = 2$ agree well with Buchalet's constrained plate data: Swedlow and Ritter's large cylinder results for $r/t = 8$ agree well with Buchalet's large cylinder data. In both cases this good agreement is more than an accident. The agreement between F_1 and F_2 tends to confirm Buchalet's suggestion that the K_I of a constrained plate can be used to approximate the K_I of a cylinder with significant bending constraint. The constraint on the cylinder can be attributed to the low value of r/t, but it produces about the same K_I as in a plate with external constraint. The second area of agreement just mentioned, between $F_1^{(C)}$ and F_8, indicates that for large values of r/t, that is, $r/t > 8$, cylinders behave as thin-walled cylinders and display a uniformly small amount of bending constraint. Finally, a comparison of the large (namely, thin-walled) cylinder results with the SEN plate results shows a further decrease in bending constraint as evidenced by the higher K_I for a SEN plate. This further decrease in constraint may be associated with the change from the doubly connected nature of the cylinder to the simply loaded plate.

Regarding Buchalet's assumption that a continuous flaw is a reasonable and less than 10 percent conservative representation of a shallow, semi-elliptical surface flaw, the comparison in Table 4 may be of interest (also includes findings of Rice and Levy[4] and Shah and Kobayashi[5]). The table lists the same parameter, F, described in Eq 8 for continuous and surface flaws in finite thickness plates. Note that a less than 10 percent difference between continuous flaws and $a/2c = 0.1$ surface flaws is indicated for values of a/t near zero. However, for flaw depths of only $0.2t$, the difference is up to 30 percent. Although both surface flaw analyses are approximate,

[4] Rice, J. R. and Levy, N., *Journal of Applied Mechanics, Transactions*, American Society of Mechanical Engineers, Vol. 39, March 1972, pp. 185–194.

[5] Shah, R. C. and Kobayashi, A. S. in *The Surface Crack: Physical Problems and Computational Solutions*, American Society of Mechanical Engineers, 1972, pp. 79–124.

TABLE 4—*Comparison of shallow flaw stress intensity factors in plates* $F = K/1.12\,\sigma\sqrt{\pi a}$.

Reference Geometry	Gross et al Continuous Flaw $a/2c = 0$	Rice and Levy Surface Flaw $a/2c = 0.1$	Shah and Kobayashi Surface Flaw $a/2c = 0.1$
$a/t = 0$	1.00	...	0.93
0.1	1.06	0.91	0.93
0.2	1.22	0.94	0.94
0.3	1.48	1.01	0.94

the fact that both indicate a significantly lower K_I for quite shallow flaws should not be ignored. This lower K_I for surface flaws could be explained by the bending constraint supplied by the uncracked material beyond the $2c$ extent of the surface flaw as opposed to the lack of such bending constraint in the case of the continuous flaw.

In defense of Buchalet's assumption, the significant difference between continuous and surface flaws in plates just mentioned might not be present in hollow cylinders due to their doubly connected nature.