APPENDIX I

THE MOTION OF CHARGED PARTICLES IN THE GEOMAGNETIC FIELD

Since the geomagnetic field is nearly a dipole field, the gross features of motions of charged particles near the earth can be determined by examining the motion of such particles in a dipole field. The Earth's field departs from that of a dipole because the currents creating the field are not confined to a simple, small, circular loop (**1,2**)[1] because of local deposits of ferromagnetic materials and because of the pressure of plasmas impinging on the field (**3,4**). At the surface of the Earth, not more than 1 per cent of the field is due to currents external to the Earth. Nevertheless, as will be seen later, a dipole representation gives a fair model of the geomagnetic field.

Motion in a Dipole Field

A particle in a homogeneous magnetic field with no electric fields present moves with a constant velocity, the component parallel to the field being unchanged. In a plane perpendicular to the field it executes a circular motion with a radius of curvature, ρ, such that

$$\rho = cm\, v_1/eB \qquad (1)$$

where:

v_1 = the component of the velocity perpendicular to the field,
m = the mass of the particle,
c = the velocity of light,
ϵ = the charge on the particle, and
B = the scalar value of the magnetic field.

The period T_1 for one revolution is given by

$$T_1 = 2\, cm/eB \qquad (2)$$

For magnetic fields similar to these around the Earth this period is typically of the order of 10^{-5} to 10^{-4} sec for electrons and 10^{-2} to 10^{-1} sec for protons.

A detailed derivation of the motion of nonrelativistic particles in nonhomogeneous and time-changing fields is given by Alfven (**5**). He shows that if the magnetic field changes very little during one revolution, certain simplifications result. The "guiding centers" approximation can be used, in which one examines the motion of the guiding centers about which the particle revolves in a motion given by Eqs 1 and 2. For most, if not all, of the particles trapped in the geomagnetic field, the guiding center approximation is valid.

In a dipole field, the motion of the guiding center consists of a rapid north-south motion along a magnetic field line and a slower longitudinal drift, eastward for negative particles and westward for positive particles. The longitudinal drift occurs along the surface of revolution of the field line along which the north-south motion occurs.

If the angle between the velocity vector of the particle and magnetic field line is α_e at the magnetic equator, then at a magnetic latitude λ, the pitch angle is given by

$$\sin^2 \alpha = \sin^2 \alpha_e (1 + 3 \sin^3 \lambda)/\cos^6 \lambda \qquad (3)$$

When the α reaches $\pi/2$, the particle is reflected; the latitude of reflection is known as the mirror point latitude, λ_m. The time between bounces is on the order of seconds or less, depending on the energy of the particle, on its pitch angle at the equator, and on the equatorial radius of the field line it moves upon.

The longitudinal drift rate in a dipole field has been calculated by several authors (**6–8**). Lew (**7**) obtains a drift period of 45 sec for a 30-Mev proton, and 87 sec for a 30-

[1] The boldface numbers in parentheses refer to the list of references appended to Appendix I, see p. 60.

Mev electron, both of which are at 2 earth radii and which mirror at the equator. For the general drift rate, he obtains the expression

$$T_3 = \frac{172.4 \text{ min } (1 + \epsilon) m_e r_e}{(2 + \epsilon) m r_0} \frac{G}{F} \quad \ldots\ldots (4)$$

where:

ϵ = the ratio of the kinetic energy of the particle to its rest energy,
m_e = the electron mass,
m = the mass of the particle in question,
r_0 = the equator radius of the field line along which the particle travels,
r_e = the radius of the earth, and
$\frac{G}{F}$ = a function of the mirror point latitude, varying from 1.0 to 1.5.

INVARIANTS OF THE MOTION IN THE GEOMAGNETIC FIELD

Northrup and Teller (9) have derived expressions for constants of the motion of particles in a magnetic field, such as that of the Earth, where these are relatively small azimuthal asymmetries. These invariants provide the best means of describing the motion of geomagnetically trapped particles. The invariants and some of their consequences are listed below:

The magnetic moment, $M = P_\perp^2/(2\ M_0 B)$, where $P_\perp$ is the component of the momentum in a plane perpendicular to the local magnetic field line. M is conserved if the electric field parallel to the magnetic belt is small. The invariance of M predicts that

$$\frac{\sin^2 \alpha_1}{B_1} = \frac{\sin^2 \alpha_2}{B_2} \quad \ldots\ldots\ldots\ldots (5)$$

and that when the magnetic field strength reaches the value $B_T = P^2/(2\ M\ m_0)$, the particle will be reflected.

The longitudinal invariant,

$$J = \oint P_{\|}\, ds,$$

where $P_{\|}$ is the component of momentum along a line of force, ds is an element of length along a line of force, and the integral is taken over a complete oscillation along the line. J is not conserved instantaneously, but its average value over a drift period is conserved if the geomagnetic field does not change much during an oscillation period, T_1. Although this condition may be violated during magnetic storms, the longitudinal invariant still offers a convenient means of describing the normal motion of typical particles. One consequence of the conservation of J is that in a static field, the shell over which the particle drifts longitudinally is closed; that is, the particle will return to the field line from which it started.

An infinite number of different longitudinal surfaces will intersect in a given field line. Thus different particles on one field line will be on different field lines at some other longitude. Northrup and Teller estimate the maximum radial difference for such intersecting shells to be about 300 km. If no electric fields are present, P is a constant and the integral variant $I = \frac{J}{P}$ is conserved. It can be shown that in this case

$$I = \oint \left(1 - \frac{B}{B_T}\right)^{\frac{1}{2}} ds$$

The flux invariant, Φ, is defined as the flux enclosed by a longitudinal invariant surface. It is conserved if the time in which the field changes is small compared to the time it takes the particle to circle the earth longitudinally. As a consequence of the invariance of Φ, during slow field changes the longitudinal invariant surface will alter its shape so as to conserve Φ.

COORDINATES FOR MAPPING TRAPPED RADIATION

If the Earth's magnetic field were that of a pure dipole, the fluxes could be mapped on a two-dimensional grid with the radial distance, r, and the geomagnetic latitude, λ, as coordinates. Because the Earth's field is nearly that of a dipole, a great deal of data have been given in these coordinates. However, at the lower altitudes, the Earth's field departs sufficiently from that of a dipole that if one uses spatial coordinates, three dimensions are required for precision. Other coordinate systems based on the invariants of the motion have been devised, however, so that only two coordinates are required. Two

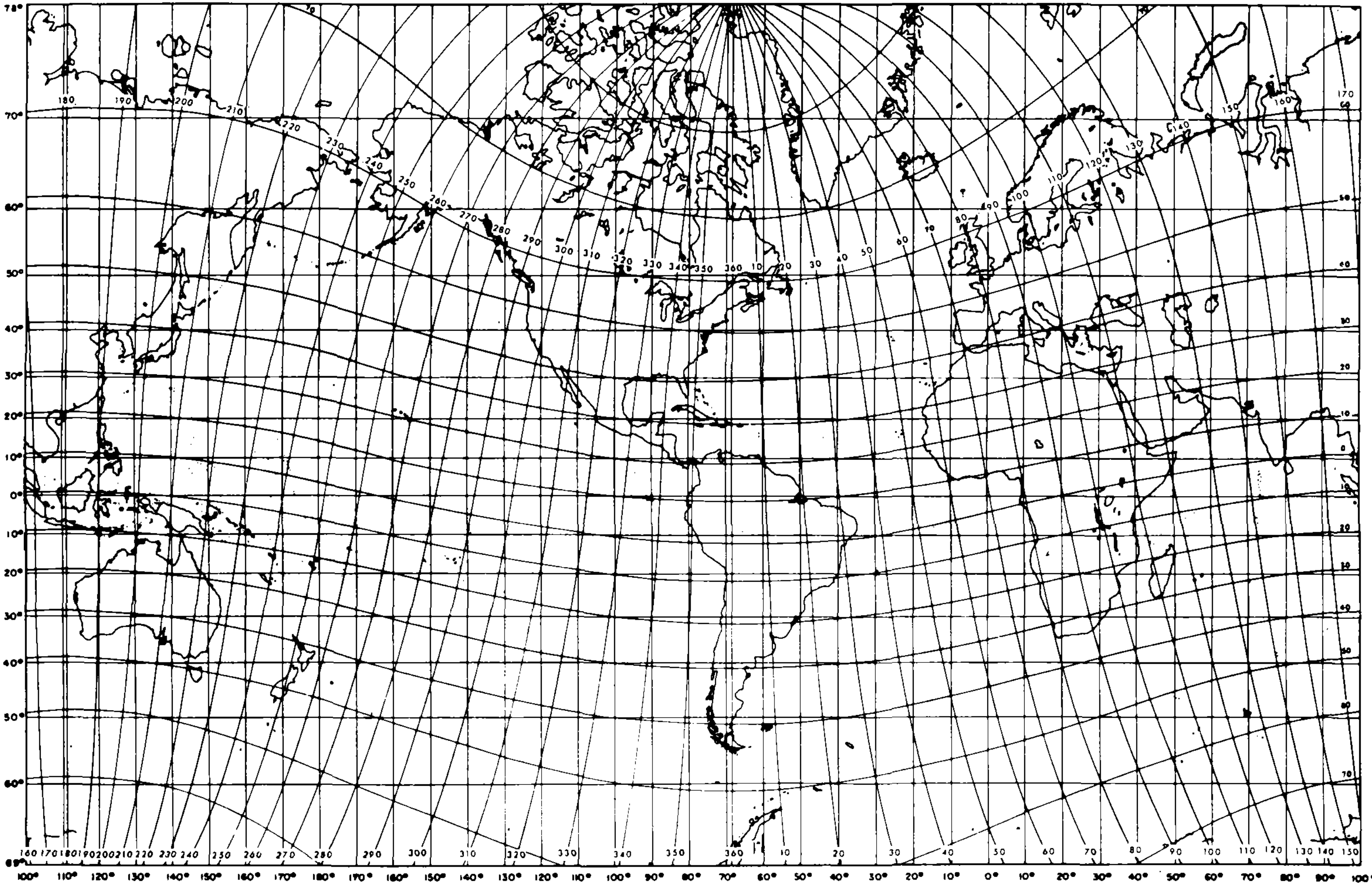

FIG. 33.—Geomagnetic Dipole Field Coordinate Grid (Curved Lines) Superimposed on a Mercator Projection of the Earth. (From reference (11) of Appendix I.)

dipole coordinate systems and a coordinate system based on the invariants are described below:

Geomagnetic Coordinates (Earth-Centered Dipole Coordinates) (**10**):

A first approximation to the geomagnetic field is a dipole located at the Earth's center but tilted at an angle to the geographic pole. Consequently, a coordinate system based on this approximation has its center at the Earth's center, but is tilted 11.7 deg toward the geographic longitude of 69.0 deg west.

integral invariant, I, the longitude-dependence would be taken care of automatically and two coordinates would be sufficient. Unfortunately, I does not have a convenient meaning. McIlwain (**13**) has shown that there exists a function, $L(B,I)$, which is very nearly constant along a magnetic field line and which is nearly equal to the radial distance to the field line at the geomagnetic equator. L retains most of the advantages of I as a coordinate for mapping trapped radiation and has the additional advantage of having an intuitive meaning. The exact

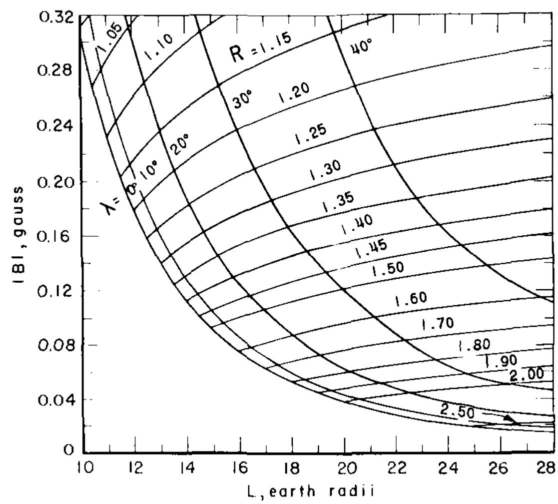

FIG. 34.—Centered Dipole Coordinates Defining B and L. (From reference (**13**) of Appendix I or reference (**27**) of Part II.)

Figure 33 gives the centered dipole coordinates on a mercator projection of the Earth (**11**).

The Eccentric Dipole (**12**):

A better fit to the geomagnetic field is obtained by approximating the field as a dipole offset from the Earth's center. The resulting coordinate system is tilted as in the case of the centered dipole, but its center is located 411.4 km from the center of the Earth on a line toward a point at 150.8 deg east and 15.6 deg north.

B and L Coordinates:

If one were to use as coordinates the scalar value of the magnetic field, B, and the

definition of L is somewhat complicated; however, for a dipole,

$$L = R/\cos^2 \lambda \quad \text{and} \quad B = M(4 - 3R/L)^{\frac{1}{2}}/R^3$$

Figure 34 illustrates this relationship.

Low-altitude data have begun to appear in the literature in B and L coordinates, often with little explanation of the coordinate system. In Figs. 29 and 30 data were presented in geomagnetic coordinates as they would appear if the Earth's field were that of a centered dipole. The dashed lines represent positions between which the Earth's surface may be at different latitudes and longitudes. It can be seen that near the equator, for a given altitude, the count rate may vary by a factor of 10^3 at different

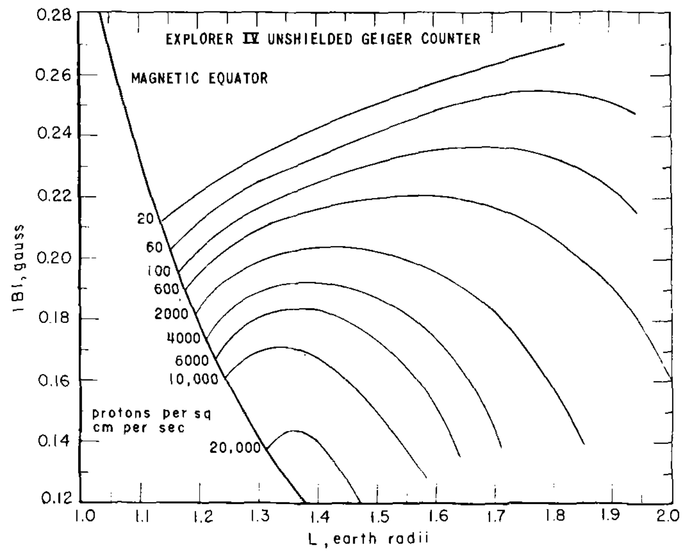

FIG. 35.—Contours of Constant Proton Flux ($E > 31$ Mev) in B and L Coordinates. These data correspond to those of Fig. 29. (From reference (**13**) of Appendix I or reference (**27**) of Part II.)

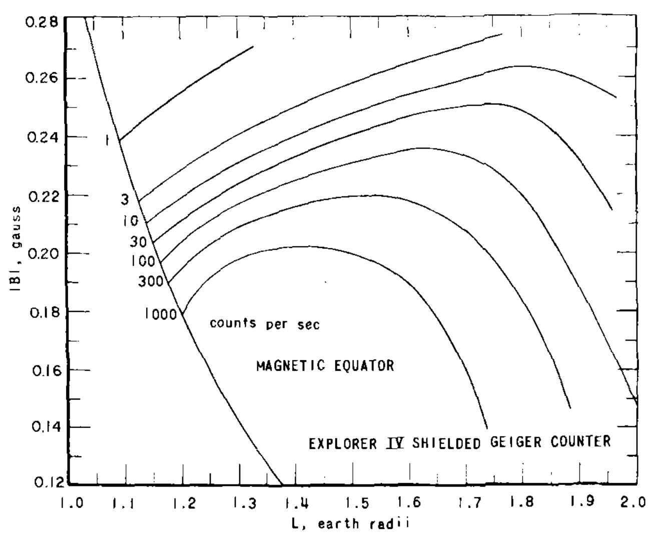

FIG. 36.—Contours of Constant Energy Flux in B and L Coordinates. These data correspond to those of Fig. 30. (From reference (**13**) of Appendix I or reference (**27**) of Part II.)

longitudes. For a given radial distance in offset dipole coordinates, the count rate may vary by as much as a factor of 10. The same data are shown in Figs. 35 and 36 as they would appear in B and L coordinates (13).

Simplified Use of B and L Coordinates:

If data to be used are given in B and L coordinates, and if high precision is sought, a complicated calculation is required. In this event one should refer to McIlwain's original article (13). However, if a factor of 10 error in the flux can be tolerated, the following procedure will usually suffice:

Let the geomagnetic coordinates be R_g, λ_g, and Φ_g, where R_g is measured from the center of the Earth and λ_g and Φ_g are the latitude and longitude. From Fig. 33 obtain the magnetic latitude, λ. Then the magnetic radius, R, is given by:

$$R = [R_g^2 + B - R_g C(0.84076 \cos \lambda_g \cos \Phi_g + 0.45449 \cos \lambda_g \sin \Phi_g + 0.26892 \sin \lambda_g)]^{\frac{1}{2}} \quad (6)$$

where $B = 169{,}250$ sq km and $C = 822.8$ km. Having R and λ, B and L can now be obtained from Fig. 34. This procedure is roughly equivalent to using offset dipole coordinates.

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APPENDIX II

ABBREVIATIONS

electron volt	ev	second	sec
thousand electron volts (10^3 ev)	kev	millimeters of mercury	mm Hg
million electron volts (10^6 ev)	Mev	neutron	n
billion electron volts (10^9 ev)	Bev	proton	p
centimeter	cm	electron	e^-
kilometer	km	positron	e^+
gram	g	alpha particle	α
kilogram	kg	beta particle	β
steradian	ster	gamma ray	γ