
ASTM Manual on
FITTING STRAIGHT LINES



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ASTM MANUAL ON FITTING STRAIGHT LINES

Prepared by

COMMITTEE E-II ON QUALITY CONTROL OF MATERIALS
AMERICAN SOCIETY FOR TESTING AND MATERIALS



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PREFACE

The ASTM Manual on Quality Control of Materials includes the presentation and treatment of data pertaining to observations on a single variable. When more than one variable is involved, statistical methods other than those discussed are required for treating the data, for presenting in a concise form the essential information contained therein, and for drawing valid conclusions therefrom.

In ASTM work a frequent problem is to discover from an examination of the observations on two variables the nature and extent of the relationship, if any, between them. Examples of pairs of variables are: the hardness of a material and its tensile strength; aging time and strength of cement; concentration of a solute and the spectrophotometric reading; assays by a standard method and by a proposed new method.

The relationship between two variables may be linear or nonlinear. The treatment of data from two nonlinearly related variables is beyond the scope of this Manual. However, such data may often be made to assume a linear, or approximately linear, relationship by suitably transforming them, as by converting each observation to its logarithm or some other appropriate function. Such transformed data if linear may be treated by the procedures here presented.

This Manual discusses some of the ways for determining when a straight line may be judged to be inadequate. It is here that a plot of the data and the fitted straight line is often helpful. This is particularly useful in identifying wild

observations that need to be rechecked. Also, if the relationship is curvilinear rather than straight line, bunching of points is very likely to occur, that is, an abnormal grouping of successive points on one side of the fitted line. It is apparent, then, that the straight line obtained by following the procedures outlined in this manual is reasonable from an engineering viewpoint only when the assumption of linearity is appropriate and is not contradicted by the data.

This manual makes no attempt at a complete survey of all the statistical methods available for treating experimental data from linearly related variables. It limits itself to a consideration of selected aspects of substantial importance in ASTM work and of frequent occurrence. Among the topics included are:

1. Presentation of the essential information in a set of linearly related pairs of observations by means of an equation (Sections 1, 2, and 3),
2. Formulas for establishing confidence limits for the constants of the line (Sections 4, 5 and 6),
3. Predictions about future observations based on the existing data (Section 7), and
4. Procedures for testing the hypotheses about the slope and intercept as well as the underlying linear relationship (Sections 8, 10, and 11).

Much of the material presented in this manual is based on Chapter 9 of *Engineering Statistics* by A. H. Bowker and G. J. Lieberman (6) © 1959 Prentice-

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This Manual was prepared by a Task Group of ASTM Committee E-11 on Quality Control of Materials. The personnel of the Task Group at the time of publication was as follows: L. Tanner, chairman, S. Collier, H. F. Dodge, R. J. Hader, G. J. Lieberman, and W. J.

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THIS PUBLICATION is one of many issued by the American Society for Testing Materials in connection with its work of promoting knowledge of the properties of materials and developing standard specifications and tests for materials. Much of the data result from the voluntary contributions of many of the country's leading technical authorities from industry, scientific agencies, and government.

Over the years the Society has published many technical symposiums, reports, and special books. These may consist of a series of technical papers, reports by the ASTM technical committees, or compilations of data developed in special Society groups with many organizations cooperating. A list of ASTM publications and information on the work of the Society will be furnished on request.

