DISCUSSION

I. K. LEE[1]—The author has suggested that "failure" of a saturated clay stressed in an undrained state will commence when the deviatric stress reaches the so called "flow limit." There are two major questions arising from the work leading to this proposition.

1. Is there any fundamental significance in the "yield value" and the "flow limit"?

2. What is the relationship of the results presented and the conclusions made to the Mohr-Coulomb failure criterion?

It is widely accepted that saturated clays exhibit both elastic and plastic components of strain at all stress levels, although the magnitude of the components is conditioned by the stress history. The yield value would therefore not represent a stress at which there is a transition from elastic behavior and the advent of significant plastic strains. The results presented by the author show that plastic strains were in fact recorded at stress levels less than the yield value. It is considered that the apparent elastic behavior at low stress levels was merely due to the insensitivity of the strain recording apparatus. Due to the progressive development of elastic and plastic strains it is difficult to accept that both the flow limit and the yield value are indicative of any change in structure of the clay. The values are arbitrarily selected points on a stress-strain curve established under specific test conditions. There is no justification for imagining that the failure deviator stress defined by the author's method is superior to the commonly accepted criterion of deviator stress at maximum effective stress ratio.

There is some difficulty in making definite conclusions from the experimental results quoted in the paper. The torsion tests, triaxial tests, and tricell tests appear to give the same order of magnitude of the yield value, but this is not surprising in view of the arbitrary nature of the parameter. The shear strains for the triaxial and tricell tests are shown in the figures but the plane over which the stress is operating is not defined (maximum shear strain?). The mean principal stress values quoted appear to be total stresses instead of effective stresses, the latter being the only satisfactory basis for comparison of the results of the three types of test.

Reference to the work of Mitchell and Campanella[2] will show that the strain behavior of saturated clays maintained in the undrained state is significantly influenced by several environmental parameters not investigated in detail by the author. The limited scope of the experiments led the author to conclude that in the "flow range" the strain rate was proportional to the difference of the deviator stress and the yield value, whereas Mitchell found that the deviator stress was proportional to the log of the strain rate. Although there is no reason at this stage to suppose that all clays

[1] Senior lecturer in civil engineering, Civil Engineering Department, Melbourne University, Melbourne, Australia.

[2] See p. 90.

obey the same law, the differences between the experimental observations could be due to the inconclusive nature of the author's experiments. The same objection can be raised to the author's statement that "the rate of flow of the strain showed itself to be independent of the sequence and shape of the stress path," but the author obviously realizes the limitations of the results by qualifying the remark.

The author is not justified to state his Eq (5) in a differential form but should quote the finite difference form, since it is apparently derived from a series of tests in which only one parameter $(\tau - f_0)$ was varied—if the statement was based on a theoretical model the use of the differential equation form is acceptable. However, integrating this equation with respect to time, regarding η as a constant which is implied by the experimental results quoted and the use of the term "coefficient of viscosity," it is seen that the total strain is proportional to time. This is in contradiction to the author's concept of the flow range.

The Mohr-Coulomb failure concept could be interpreted either on a total stress or an effective stress basis. The relationship of the author's investigations to the proof of the applicability of the concept is difficult to understand. Certainly no reference is made to an effective stress study. The writer is at a loss to accept that Mohr-Coulomb is invalid because failure can occur at a deviator stress less than the maximum value, which appears to be the author's major criterion. Surely the procedure for establishing the validity of Mohr-Coulomb, or any other failure hypothesis, is to determine whether the "frictional" and "cohesive" parameters of the equation are in fact dependent on the stress path leading to failure—where failure is the accepted definition such as peak principal stress ratio or perhaps the author's.

The writer finds it even more disturbing that the author states that the "concept proves to be valid for granular systems within a certain range of densities" (medium and high density quoted in conclusions). The three criteria for acceptance as stated by the author are not sufficient for the purpose of establishing the validity of the concept. The only basis is an investigation of the type mentioned above (it is also noted in passing that the third "requirement" is not true for dense sands at low stress levels).

The frictional parameter defined by Mohr-Coulomb and measured in drained tests on sands includes the effect of energy of dilation as well as the fundamental frictional resistance to particle movement plus a component due to remolding of the specimen. Rowe[3] has presented very extensive experimental proof that the behavior of sands under triaxial deviatric stress conditions can be expressed by a criterion similar in mathematical form to Mohr-Coulomb but taking into account the energy of dilation. The feature of relevance in this discussion is that the most significant departure from Mohr-Coulomb occurs with dense sands. The best agreement is obtained with saturated clays tested under an undrained condition.

E. C. W. A. GEUZE (*author*)—Mr. Lee repeatedly uses the adjective "plastic" where the material behavior clearly is of a viscous nature. He also overemphasizes the significance of the very small permanent strains below the yield value resulting from a small air content of the specimen, and makes a presumptuous suggestion concerning the lack of sensitivity of the apparatus in order to explain

[3] P. W. Rowe, "The Stress-Dilatancy Relation for Statical Equilibrium for an Assembly of Particles in Contact," *Proceedings*, Royal Society, A, Vol. 269, 1962, pp. 500–527

the apparent elastic behavior of the clay system.

The yield value and the flow limit have not been obtained by an arbitrary selection of points on a stress-strain curve. They appear in the right-hand diagram of Fig. 5 as a result of the procedures used in elasticity and in rheology to define the coefficients G and η. The results of the stress-strain-time curves in Fig. 4 have been used for this diagram. The yield value and the flow limit then appear.

I should be interested to know what test results are not obtained under specific conditions. It is fortunate that most investigators do specify testing conditions, which renders a discussion of their results meaningful.

I have made a statement concerning the indeterminate nature of "failure" deviator stress (Conclusion 6). There is therefore no justification for Mr. Lee's statement that I "imagine" this quantity to be superior to any other "commonly accepted criterion." A statement of this kind simply does not appear in the paper.

I should point out that I do realize the significance of pore pressure, but I have not found as yet sufficient evidence to establish a unique relationship between the maximum effective stress deviator and the failure condition of a saturated clay.[4]

Mr. Lee is correct in his assumption that the mean principal stress values are total stresses. Furthermore, in an isotropic material (saturated, remolded clay) the relationship between the stress tensor and the strain tensor is sufficiently established in the elastic range and in the flow range. This makes the definition of "the plane over which the stress is operating" superfluous.

Mr. Lee refers to the work of Mitchell and Campanella presented in this symposium. I am not in the habit of discussing unpublished material in the closure of a discussion of my own paper. Since Mr. Lee prefers, however, to quote "several uninvestigated, environmental parameters" as the source of the discrepancy between the strain rates found at a constant deviator of stress in the studies of Mitchell and Campanella and in my paper, it is only fair to correct this erroneous impression. The logarithm of the creep rate was found to be proportional to the deviator of effective stress in the tests of Mitchell and Campanella, whereas I found the total deviator of stress to be proportional to the creep rate. In both investigations the specimen temperature was maintained rigorously constant. It may also be of interest to Mr. Lee to study the results of my investigations on flow phenomena of drained specimens[4] and the references to this study by Hvorslev.[5]

Mr. Lee's objection to the statement that the rate of flow is independent of sequence, shape, and time scale of the stress path, is based on a quotation out of context of the relevant paragraph of my paper. Not only did I specifically state the limited time scale as the reason for the observed behavior in these test series, but I considered this point of sufficient importance to be entered as Conclusion 10 of my paper.

Mr. Lee's discussion of the use of partial derivatives in Eq (5) appears to be based on a lack of familiarity with the theory of rheology. Treatment of this question in textbooks on this subject bring one to the conclusion that the total strain is not proportional to time

[4] E. C. W. A. Geuze, "Horizontal Earth Pressure Against a Row of Piles," *Proceedings*, Second International Conference on Soil Mechanics and Foundation Engineering, Vol. IV, 1948, pp. 135–140.

[5] M. J. Hvorslev, "Physical Components of the Shear Strength of Saturated Clays," *Proceedings*, Am. Soc. Civil Engrs. Conference on Shear Strength of Cohesive Soils, 169 June, 1960 p. 220.

with varying magnitudes of $(\tau - f_0)$, and this conclusion is not in contradiction with my concept of the flow range.

The Mohr-Coulomb failure concept should be interpreted in terms of effective stresses. For this reason, my statements concerning the uniqueness of the concept were deliberately limited to granular systems. More information is given in Ref (3)[6] of my paper, where granular systems in effective states of stress were used exclusively.

collapses with increasing values of the deviator of effective stress to be incompatible with my first requirement. However, a sufficiently wide range of densities still exhibits the required characteristics to satisfy the Mohr-Coulomb concept, amongst these the proof of the fact that the frictional parameter is independent of the stress path to failure.

It should also be noted—though not in passing—that I am very familiar with the phenomenon of the interlocking

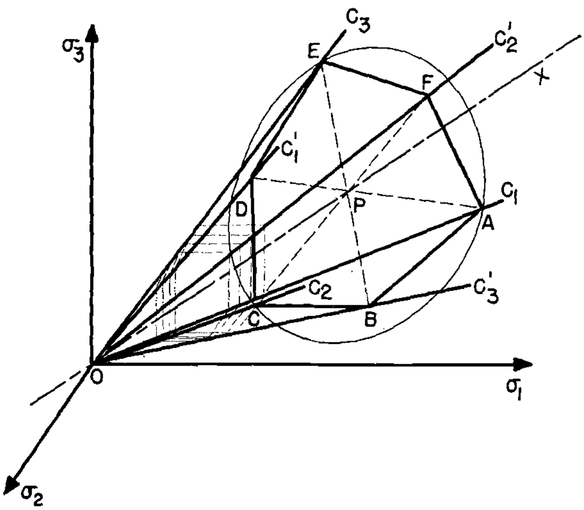

FIG. 6—Mohr-Coulomb Failure Surface in a Tridimensional Stress Space for Noncohesive Materials.

The Mohr-Coulomb failure concept can not be interpreted in terms of total stresses for reasons clearly stated in my paper.

On the question of why I exclude loose structures of granular systems from the applicability of the Mohr-Coulomb concept, I refer Mr. Lee to Ref (3) in the paper. The relevant investigations clearly show the sequence of local structural

resistance which makes itself felt at very low stress levels in granular systems. My investigations, however, have shown that the order of magnitude seldom exceeds 10 gr/cm² and therefore can be neglected for practical purposes. In order to satisfy even minor objections I added to the definition of the third requirement, in the form of Eq (2), the following specification: "when the interlocking resistance at zero normal stress is a negligible quantity."

His final comments indicate that Mr. Lee may have missed several points in

[6] E. C. W. A. Geuze, "Critical Density of Some Dutch Sands," *Proceedings*, Second International Conference on Soil Mechanics and Foundation Engineering, Vol. IV, 1948, pp. 125–130.

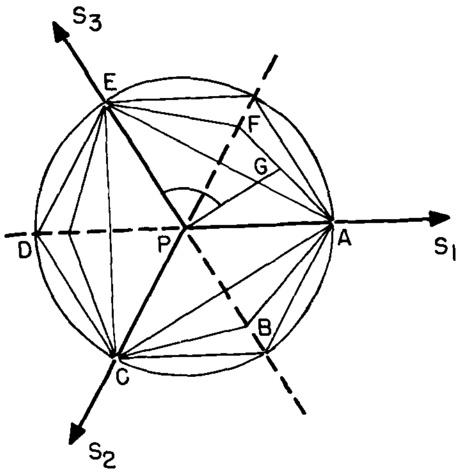

FIG. 7—Section of Mohr-Coulomb Failure Surface by an Octahedral Plane.

my paper. I specifically stated that the Mohr-Coulomb concept of the state of failure is a stress criterion excluding energy considerations. A study of the original contributions of both Mohr and Coulomb would undoubtedly lead Mr. Lee to the same conclusion.

I appreciate Mr. Lee's comments insofar as he has drawn attention to the need for clarification of a number of points in my paper. He has, however, failed to substantiate his stated relevant features.

ADEL S. SAADA[7]—The following discussion deals only with that part of the paper related to noncohesive soils. Since the Mohr-Coulomb concept in shear failure is fundamental, it seems appropriate in this discussion and in the frame of this symposium to establish the case for (or against) the validity of this concept. This will be done while examining the points which seem to have led the author to make his statement about the uniqueness of its applicability. The following analysis of the space and plane representation of Coulomb's law will be helpful to the reader of the vaiious papers referred to in this discussion.

In a stress space, the Coulomb surface of failure for an isotropic material is

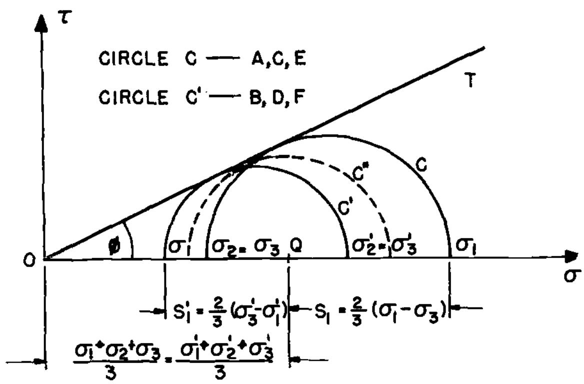

FIG. 8—Mohr Representation of Various Points on Fig. 7.

represented by a slender hexagonal pyramid whose apex falls at the origin. Since the intermediate principal stress, σ_2, is assumed to have no effect on the values of σ_1 and σ_3 at failure, the six planes of this surface are generated by segments of straight lines parallel to the principal axes (Fig. 6). These planes intersect along the straight lines C_1, C_2, C_3, C_1', C_2', and C_3' meeting at the origin. A section of this surface by an

[7] Assistant professor of engineering, Case Institute of Technology, Cleveland, Ohio.

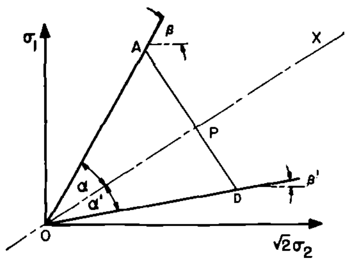

FIG. 9—Section of Failure Surface by a Plane Through σ_1, the Line $\sigma_2 = \sigma_3$, and the Space Diagonal OX.

represented by a circle between C and C' and tangent to OT. The hydrostatic component of the state of stress for all these circles is the same (point Q in Fig. 8). As an example, a state of hydrostatic stress coupled with torsion is represented by point G (or by equivalent points on AB, BC, $\cdots$) in Fig. 7 and by circle C'' centered at Q in Fig. 8. A change in the state of hydrostatic stress gives another hexagon and another cluster of circles

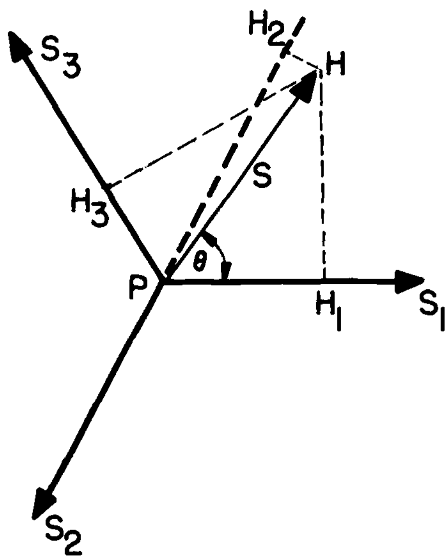

FIG. 10—Location of Experimental Point on Octahedral Projection.

octahedral plane (Figs. 6, 7) gives the well-known hexagon corresponding to a given hydrostatic stress and a given angle of internal friction. The knowledge of one of the sides of the hexagon is sufficient. When this hexagon is translated to the (σ, τ) system of coordinates, Fig. 8 is obtained (for one given state of hydrostatic stress, σ_m). For an isotropic material, circle C corresponds to points A, E, C of Figs. 1 and 2, and circle C' corresponds to points D, B, F. Any point on line AF (or AB, BC, $\cdots$) can be on the (σ, τ) diagram. This new cluster of circles is, of course, still tangent to OT in Fig. 3. A section of the surface by a plane passing through σ_1 and the line $\sigma_2 = \sigma_3$ is shown on Fig. 9. From Figs. 6, 7, 8, and 9, the following geometrical relations can easily be established:

$$\frac{PA}{PD} = \frac{PE}{PB} = \frac{PC}{PF} = \frac{3 + \sin \phi}{3 - \sin \phi} \quad \ldots\ldots (1)$$

$$\tan \alpha = \frac{4 \sin \phi}{\sqrt{2}(3 - \sin \phi)} \quad \ldots\ldots (2)$$

$$\tan \alpha' = \frac{4 \sin \phi}{\sqrt{2}(3 + \sin \phi)} \quad \ldots\ldots (3)$$

$$\tan \beta = \frac{1 + \sin \phi}{\sqrt{2}(1 - \sin \phi)} \quad \ldots\ldots (4)$$

$$\tan \beta' = \frac{1 - \sin \phi}{\sqrt{2}(1 + \sin \phi)} \quad \ldots\ldots (5)$$

To place an experimental point on the octahedral section corresponding to a given hydrostatic stress σ_m, one usually subtracts σ_m from the principal components of the stress tensor to obtain the principal components of the deviator tensor S_1, S_2, S_3 (Fig. 8). The lengths PH_1, PH_2, PH_3, are then taken on the projection of the principal axes on the octahedral plane so that (Fig. 10)

$$PH_1 = \sqrt{\frac{3}{2}}\, S_1 \quad \ldots\ldots (6)$$

$$PH_2 = \sqrt{\frac{3}{2}}\, S_2 \quad \ldots\ldots (7)$$

$$PH_3 = \sqrt{\frac{3}{2}}\, S_3 \quad \ldots\ldots (8)$$

The three normals from H_1, H_2, and H_3 meet at H, which represents the state of stress deviation.

To support the statement that σ_2 does not affect the extreme values of σ_1 and σ_3 at the point of failure, the author refers only to the investigation of Kirkpatrick.[8] This investigation showed that for a practically uniform sand, the angle of internal friction in triaxial compression, ϕ_c, was equal to that in triaxial extension, ϕ_s. Also, variations of σ_2 between σ_1 and σ_3 gave for the same material values of ϕ higher than ϕ_c by no more than 2 deg. To support the statement that the envelopes of Mohr circles corresponding to the major and minor principal stresses are straight, Dr. Geuze refers without comment to numerous triaxial failure tests.

The available literature on the subject is limited and is far from showing unanimous agreement with Kirkpatrick's results. It can be easily reviewed in brief:

Kjellman[9] ran tests on a uniform sand by means of a cubical box which allowed him to change σ_1, σ_2, and σ_3 independently. The angle of friction deduced from compression tests ($\sigma_1 > \sigma_2 = \sigma_3$) was 35 deg, while the one deduced from tests with an intermediate value of σ_2 ($\sigma_1 > \sigma_2 > \sigma_3$) was 43 deg.

Bishop and Eldin[10] ran tests on the medium-to-fine fraction of a well-graded sand and found the same value for the angle of internal friction whether deduced from triaxial compression or triaxial extension tests.

Habib[11] in his doctoral dissertation ran both extension and compression tests as well as torsion tests on solid sand cylinders of various granulometry. He found that in all cases the angle of internal friction deduced from compression tests was higher than the one deduced from tension tests by an amount varying between $6\frac{1}{2}$ and 9 deg. From torsion tests, he was able to obtain failure for different values of $\sigma_1 > \sigma_2 > \sigma_3$ and study the influence of the intermediate principal stress on ϕ. He found differences up to 11 deg for various stress combinations. For the same tests no

[8] W. M. Kirkpatrick, Discussion of "Mechanics of the Triaxial Test for Soils," *Proceedings*, Am. Soc. Civil Engrs., April, 1961, p. 175–177.

[9] W. Kjellman "Report on an Apparatus for Consumate Investigation of the Mechanical Properties of Soils," *Proceedings*, First International Conference on Soil Mechanics and Foundation Engineering, Cambridge, Vol. I, 1936, pp. 16–20.

[10] A. W. Bishop and A. K. G. Eldin, "The Effect of Stress History on the Relation Between φ and Porosity in Sand," *Proceedings*, Third International Conference on Soil Mechanics and Foundation Engineering, Zurich, Vol. I, 1953, pp. 100–105.

[11] P. Habib, "La Resistance Au Cisaillement Des Sols," Thesis, University of Paris.

correlation could be made between the inclination of the helicoidal surface of failure and the inclination predicted by Mohr-Coulomb failure criterion. ($\pi/4 - \phi/2$ with σ_1).

Peltier[12] assembled on the same figure the results of Habib, Brice, and those of the "Laboratoire des Ponts et Chaussées" and noticed that, as an average, ϕ_c was higher than ϕ_s by about 7 deg. He designed a triaxial box and tested various sands of uniform granulometry. In his case, ϕ_s was found to be larger than ϕ_c.

Haythornthwaite[13] ran a series of tests on a uniform quartz and obtained results supporting Habib's conclusions that $\phi_c > \phi_s$. He proposed a failure criterion based on a triangular shape (Fig. 6) instead of a hexagon and expressed the equations of the three planes forming the surface of failure by means of a cohesion (when treating the general case) and an angle of friction as determined from triaxial compression. It is clear that in this case ϕ and C have very little, if anything, to do with Coulomb's criterion. They become two experimental parameters necessary to determine the triangle.

Wu, Loh, and Malvern[14] ran compression and tension tests as well as tests on hollow cylinders extending Kirkpatrick's investigation. They found good agreement between φ_c and φ_s; but a variation of σ_2 between σ_1 and σ_3 resulted in an angle of internal friction 5 deg higher.

[12] M. R. Peltier, "Recherches Experimentales sur la Courbe Intrinseque de Rupture des sols Pulverulents," *Proceedings*, Fourth International Conference on Soil Mechanics and Foundation Engineering, London, Vol. I, 1957, pp. 179–182.

[13] R. M. Haythornthwaite, "Mechanics of the Triaxial Test for Soils," *Proceedings*, Am. Soc. Civil Engrs., SMf Division, Vol. 86, No. SM5, 1960, pp. 35–61.

[14] T. H. Wu, A. K. Loh, and L. E. Malvern, "Study of Failure Envelope of Soils," *Proceedings*, Am. Soc. Civil Engrs., SMf Division, No SM1. Paper 3430, February, 1963, pp. 145–181.

In the previous investigations, the peak point was chosen by Bishop and Eldin, Kirkpatrick, and presumably by Habib and Wu to define failure. Haythornthwaite used a modulus to compare tension and compression. Peltier did not give any indication about his definition of failure: thus, a comparison of results seems difficult, if not impossible. However, a question that can be asked is: "To what extent can results obtained from extension and compression tests be considered simultaneously with those obtained from other types of tests ($\sigma_1 \neq \sigma_2 \neq \sigma_3$) in the process of checking the validity of Coulomb's law?" J. Biarez[15] has shown that, if a specimen is initially isotropic and if during testing the deformation occurs along a straight line of constant direction in the principal deformations space (simple path), the peak strength can be represented in a tridimensional principal stress space. Examples of a simple path are provided by extension and compression tests. On the other hand, if the deformation (or the state of stress) is subject to changes in direction on the path to failure, large structural anisotropies (also noticed by Peltier in his experiments) occur in the material and the peak strength can no more be represented in a tridimensional stress space. Six dimensions would probably be needed. Consequently, Biarez points out that if the peak strength is chosen to determine the surface of failure, results of compression and tension tests cannot be considered simultaneously with results obtained from torsion tests. He remarks, however, that such a consideration can be made if one chooses to plot the values corresponding to the ultimate strength to determine the failure surface (Fig.

[15] J. Biarez, "Contribution a L'Etude des Proprietes Mecaniques des sols et des Materiaux Pulverulents," Thesis, Universite de Grenoble.

11). This ultimate is reached when the stress-strain curve becomes parallel to the strain axis and no volume changes occur in the specimen. The state attained is thus similar to a state of perfect plasticity. At this stage one can assume that the deformation tensor keeps the same direction and moves along a simple path. The magnitude of the deformation is large enough to erase the past history of the material. Biarez's analysis could explain the differences between the results of Kirkpatrick and Wu whose tests are essentially of the same nature but with different stress paths. These

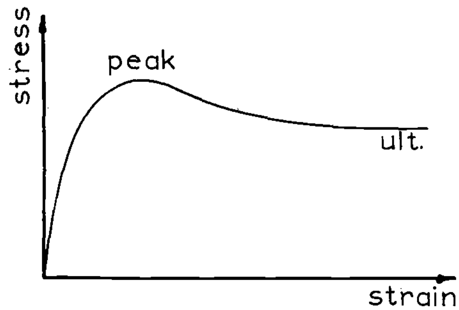

FIG. 11—Typical Stress-Strain Curve.

findings are also a highly probable explanation for the large differences between the results of Kirkpatrick on one hand and those of Habib and Peltier on another.

The problem of nonuniform stress distribution is often referred to whenever tests do not give the results predicted by Coulomb's law. I think that it is somewhat unfair to advance it as a principal reason for the failure of the efforts to identify the envelope of Mohr's circles with the directions of observed slip planes in conventional tests, as is done by the author. The conventional triaxial compression test is the only one which gives nearly consistently a straight envelope passing through the origin for the various values of the mean stress. Its results are not questioned or modified to take into account these nonuniformities in the stress distributions at failure. They are plotted directly on a (σ, τ) diagram, and they provide the straight-line envelope which is one of the very few, if not the only, strong asset in favor of Coulomb's law of failure. The question is, why are these nonuniformities brought forward to justify the invalidity of half of the results, namely, the failure along planes different from those predicted by Coulomb's law, and entirely ignored when the other half of the results, namely, the numerical values of the principal stresses at failure, coincides with the predictions of the same law? In the absence of any evidence, it appears difficult to conceive that these nonuniformities influence the inclination of the failure planes to such a high degree and the magnitude of the stresses at failure to such a low one that the latter can be comparatively neglected.

In view of the available literature and the experimental data, which are far from being conclusive, I find it difficult to accept without further evidence the statements that have led the author to conclude the uniqueness of the applicability of the Mohr-Coulomb concept to noncohesive soil. I hope that this discussion, together with Dr. Geuze's closure, will help define the actual status of this failure condition.

DR. GEUZE (*author*)—Dr. Saada presents his discussion in four distinct parts.

The first part deals with the representation of the Coulomb failure condition in a stress space. This material has been published previously by several authors[16,17] and does not present any new viewpoints.

The second part contains references

---

[16] A. Nadai, *Theory of Flow and Fracture of Solids*, Vol. I, McGraw-Hill, New York, N. Y., 1950, pp. 175–228.

[17] O. Hoffman and G. Sachs, *Introduction to the Theory of Plasticity for Engineers*, McGraw-Hill, New York, 1953, pp. 40–43.

to investigations of the effect of the intermediate principal stress on the ratio of the extreme values of the principal stresses at failure. Dr. Saada discusses these results and arrives at the conclusion that comparison of the results seems difficult, if not impossible. I agree with this statement to the extent that results obtained by various experimental methods often are contradictory. This does not mean, however, that these results necessarily are of equal value. It is obvious that in the hollow cylinder design, as used by Kirkpatrick, the tensor of the principal strains coincides with the tensor of the principal stresses practically throughout the entire cylindrical space. For this reason alone his test results are therefore to be preferred in discussions concerning the effects of the intermediate principal stress on the extreme principal stress ratio.

At the time of writing this paper, I was unfortunately not acquainted with the paper published by Wu, Loh, and Malvern. I have now had an opportunity to study this paper and also Dr. Haythornthwaite's discussion,[18] which concurs with my own views on the subject.

In the third part, Dr. Saada has questioned my sources of information concerning the straightness of Mohr circle envelopes. I hope to be relieved of the cumbersome task of producing several hundreds of my own test results or those of other investigators, which are abundantly available in the literature, to prove this point.

It is not quite clear what Dr. Saada means by his question: "To what extent can results obtained from extension and compression tests be considered simultaneously with those obtained from other types of tests ($\sigma_1 \neq \sigma_2 \neq \sigma_3$) in the process of checking the validity of Coulomb's law?" If this question is interpreted in the following manner: "Should tests with $\sigma_1, \sigma_2 = \sigma_3$ and $\sigma_1 = \sigma_2, \sigma_3$ be compared with those where $\sigma_1 > \sigma_2 > \sigma_3$ in order to check the validity of Coulomb's law?," the answer can only be in the affirmative.

With respect to Dr. Saada's comments on a study by J. Biarez concerning the state of perfect plasticity reached at the point of ultimate strength, he is referred to a study by G. de Josselin de Jong.[19] My tests on the effect of the stress path[20] have shown that the value of ultimate strength is hardly affected, even when such extremes are applied as an initial value of the hydrostatic component of stress equal to the minor principal stress, or an initial value equal to the largest principal stress. The strains preceding the state of ultimate strength, however, are strongly affected by these different stress paths.

It is therefore highly improbable that different stress paths would explain the differences between the results obtained by Kirkpatrick and by Wu, et al. on hollow cylinders.

When comparing these results with those obtained by Kirkpatrick, Wu, et al., Bishop and Eldin, de Josselin de Jong, and me on compression and extension tests, there is strong evidence that the "intermediate" effect is not significant. This was the statement in my paper.

In making his final statements in the fourth part of his discussion, Dr. Saada seems to be under a serious misapprehension of the basic meaning of Cou-

[18] R. M. Haythornthwaite, Discussion of paper by T. H. Wu, A. K. Loh, and L. E. Malvern, entitled, "Study of Failure Envelope of Soils," Journal of Soil Mechanics and Foundation Division, *Proceedings*, Am. Soc. Civil Engrs., 89, SM5, September, 1963, p. 119.

[19] G. de Josselin de Jong, "Statics and Kinematics in the Failure Zone of a Granular Material," Doctorate Thesis, Delft University, Holland, 1959.

[20] E. C. W. A. Geuze, "Critical Density of Some Dutch Sands," *Proceedings*, Second International Conference on Soil Mechanics and Foundation Engineering, Vol. IV, 1948, pp. 125–130.

lomb's law. On more than one occasion I have observed the nonuniformity of the strain distribution inside specimens loaded to the point of their ultimate strength. This nonuniformity excludes identification of the slip-line field boundaries with the $(\pi/4 - \phi/2)$ value. The stress distribution at the point of ultimate strength is governed by a nonuniform state of strain as, for instance, caused by the dilatant properties of the granular material and the kinetics within the field of failure.

The fact that Coulomb's law still applies under these conditions is due to its uniqueness, which prescribes the ultimate frictional resistance along the boundaries of the slip-line field and the rigid portions of the medium. Whatever the stress distributions are along these boundaries, the proportionality between the shearing resistance and the applied forces will be maintained.

This proportionality is the essence of Coulomb's law, which, as Dr. Saada agrees, is its only strong asset.

I am also glad to observe at this point that Dr. Saada agrees to the near consistency of the straight envelope passing through the origin of the s-σ diagram.

After this, I can not see that there is another half of Coulomb's law, ignored in predicting the extreme values of the principal stresses.

Coulomb's law can not predict these values for the simple reason that the parameter ϕ can be obtained only by experimentation. What can be predicted is the fact that ϕ will be a constant, that is, that the ratio of the principal stresses is a constant.

RONALD F. SCOTT[21]—The problem of determining the shape of the yield surface for soils in principal stress space is a difficult one, and, while respecting the author's judgment, I find it difficult to agree with him on the basis of the evidence so far presented that the intermediate principal stress has only a small effect on the stress conditions at yield in a granular medium.

It is difficult to compare quantitatively results obtained from different tests on specimens with various shapes, and, indeed, another paper by Roscoe, Schofield, and Thurairajah[22] in the present symposium tackles the difficulty of even comparing triaxial compression and extension tests. When the yield surface in principal stress space is described by the Mohr-Coulomb criterion, the descriptive constants are the well-known "cohesion" and "angle of internal friction" of the soil, to which attempts have been made to attach physical significance. If another yield equation must be adduced to describe measured data or develops from theoretical considerations, it will involve new constants for which different names will be required. Thus, it is not, strictly speaking, proper to discuss the present question in terms of whether or not the angle of internal friction of a soil is the same in the triaxial compression, triaxial extension, and plane strain tests, although it is convenient to do so, since the friction angle is a readily identifiable quantity.

If failure in, for example, a cohesionless material is observed in a particular test in which a given stress path to yield in principal stress space is followed and a yield surface based on the Mohr-Coulomb failure theory is constructed to pass through the observed yield point in stress space, the descriptive constant required for the Mohr-Coulomb surface is an angle of internal friction which is then associated with the observed yield point of the material, regardless of the actual yield surface equation.

[21] Associate professor of civil engineering, Division of Engineering and Applied Science, California Institute of Technology, Pasadena, Calif.

[22] See p. 111.

To examine the deformational behavior and yield surface of soil, a testing apparatus has been built at the California Institute of Technology by a graduate student, J. M. Bell, working under my supervision. The equipment is capable of subjecting a soil specimen to three differing controlled principal stresses and enables the principal and volumetric strains to be measured. In tests carried out to date on an Ottawa sand in a dense state at constant hydrostatic stress, a yield envelope has been consistently observed which does not conform to the Mohr-Coulomb criterion but lies closer to the circle of the Von Mises yield envelope. In terms of the "angle of internal friction" as discussed above, this implies higher "friction angles" in tests corresponding both to triaxial extension and plane strain than in those corresponding to triaxial compression.

Since bearing capacity factors are nonlinearly dependent on the "angle of internal friction" and may differ by factors of two or three for changes in the "angle of friction" of only a few degrees, the importance of properly establishing a yield surface or equation follows.

When a yield surface is established, the question of selecting a simple practical test or tests with which to locate the suitable (possibly new) constants for test specimens of soil from the field must also arise, together with the accompanying mathematical consideration of obtaining solutions to practical problems involving yielding of soils.

DR. GEUZE (*author*)—Dr. Scott clearly prefers the yield surface in the principal stress space as a basis for a failure concept of a granular medium. In view of the present sufficient lack of experimental evidence, I do not see any objection to the use of the angle of internal friction to describe the yield condition of a granular medium. The physical significance of friction as the major source of strength of granular systems has been sufficiently established ("cohesion" has to be rejected as a physical magnitude since in the various tests quoted by Dr. Scott other factors than the magnitude of the friction interfere with the test results). I have however indicated in my paper that under extreme and intermediate conditions of the stress paths the results do not differ significantly. My observations have been confirmed by several investigators.

I regret that present tests do not provide the information required to substantiate the yield surface, which would successfully replace the Mohr-Coulomb criterion. I am however interested in the tests currently performed at the California Institute of Technology, since similar efforts to obtain a uniform medium subjected the three differing controlled principal stresses have mostly failed.

I disagree, however, with Dr. Scott, that a difference of only a few degrees (and subsequent variations in bearing capacity factors) warrant the efforts required for conclusive evidence concerning the yield surface. Density variations of natural or man-made granular deposits do affect the magnitude of the angle of internal friction very strongly. The practical significance of the bearing capacity factors is questionable, not only because of these density variations, but also because of insufficient proof of the validity of the basic theories on the bearing capacity itself.

For the same reason a simple, practical test or tests would suffer from the same deficiencies, which are the cause of our present predicament, and their results (constants) would be out of step with the (presumed) accuracy of the mathematical considerations with which the solution to practical problems would be obtained.