DISCUSSION

B. O. CORBETT[1]—The paper by A. W. Bishop and R. E. Gibson shows theoretically that in the triaxial test the coefficient of consolidation can be determined only in clays of low permeability when filter strips are used. Even in this case the surface area of the specimen over which the drains are effective is important.

In the laboratory of Soil Mechanics Ltd. a number of consolidated-undrained triaxial tests have been carried out concurrently and on the same specimens as triaxial consolidation tests. Filter drains were used for the shear tests, and it has been assumed for the purposes of calculation that the drains are fully effective. Drainage was from the porous disk at one end of the specimen only in all tests. A comparison of the results of the shear and consolidation tests is given in Fig. 10.

The porous stones used were of the ceramic UNI A80 KV, and the filter drains of Whatman 54 filter paper. The specimens were of very weathered rock from Nigeria.

All specimens were saturated by back pressure to a final criterion of a B value not less than 0.96 for a 10 psi increment in cell pressure. The results are presented in terms of the coefficient of permeability, k

$$c = \frac{k}{wm_v}$$

The values have been taken over comparable pressure ranges such that m_v is sensibly constant and therefore, $k \propto c_a$.

Fig. 10 shows the following:

1. The coefficient of consolidation can be determined when filter drains are used only where the soil is less permeable than 2×10^{-9} cm/sec. The drains are not fully efficient and indicate that consolidation occurs at about half the rate of that expected for drains covering the whole surface of the specimen.

2. The drains do not assist consolida tion when the permeability of the soil is greater than about 3×10^{-6} cm/sec.

The results substantiate the theories developed by the authors and would indicate that there is little point in using filter drains in the triaxial test for soils more permeable than, say, 10^{-6} cm/sec.

However, this argument applies to saturated soils. The first stage of many triaxial tests is to saturate the specimen by the application of back pressure. The soil in its initial state may exhibit a significantly lower permeability than when saturated. It is at this stage of the test that filter drains are useful, and even though they may be unnecessary during consolidation (and shearing), their use enables the total time taken for a test to be reduced for soils more permeable (when saturated) than 10^{-6} cm/sec.

It should not be forgotten that (1) filter drains do affect the strength of the soil measured in the triaxial test, and (2) there is an increased risk of trapping air between the membrane and the specimen when filter drains are used.

RONALD F. SCOTT[2]—The paper is an interesting analysis of a very complex

[1] Senior engineer, Soil Mechanics Limited, London, England.

[2] Associate professor of civil engineering, Division of Engineering and Applied Science, California Institute of Technology, Pasadena, Calif.

problem, and the writer compliments the authors for taking the trouble to express their results in compact graphical form. In connection with Eq (1), the authors say that a test giving results at variance with this equation "... might suggest that the soil skeleton exhibited time effects not accounted for in Terzaghi's theory." An observed departure from a curve described by a solution to the usual linear soil consolidation equation may occur, as the authors mention in the case they are treating, owing to the hydraulic resistance of drainage materials, but also may arise both from the possible non-linear behavior of the soil skeleton, which need not be time-dependent, and from boundary constraints placed upon the soil specimen's deformation.

The deviations from a theoretical curve may therefore be ascribed to material or geometrical causes, a fact the authors are well aware of by subsequent remarks in the paper. In a previous paper[3] and closure, the writer considered the problem of a variation in the coefficient of consolidation and the effect of the variation on the interpreted value of the coefficient based on an assumption of constancy.

It is the purpose of this discussion to consider briefly a geometrical effect in the triaxial test and to compare the solution to the authors' results. Were the triaxial soil specimen free to do so, it would, in the course of consolidating according to the boundary drainage conditions which give rise to the authors' solutions, pass from its original prismoidal form in which its boundaries have straight-line generators, through intermediate stages of convex boundaries, to a final shape also generated by straight lines (providing its deformation properties were not path-dependent). In fact the circular surfaces are constrained to remain plane and parallel, although no such constraint is placed on the cylindrical surface. If the assumption is made that the deformation process can be considered to be one of equal vertical strain, ignoring the lateral restraints at the top and bottom surfaces, another solution to the consolidation problem can be obtained, in which the material is, however, considered to behave in accordance with Terzaghi's theory. Solutions of the equal strain type were first obtained by R. A. Barron[4] for the problem of the consolidation of clays by drain wells.

A solution was obtained to the equal vertical strain case for the triaxial test specimen under the following conditions. The specimen was considered to be surrounded by a permeable filter-paper drain as in the authors' case, but since it presented little more difficulty in solution, the presence of a smeared layer of soil next to the drain was also taken into account. Rather than ascribe a finite thickness, permeability, and compressibility to this layer, its function was considered to be only one of resistance to the flow of water through it, so that the relation

$$k_r \frac{\partial u}{\partial r} + K(u - v) = 0 \ldots\ldots (48)$$

was considered to hold at the boundary $r = a$, where K is an hydraulic transfer coefficient between the undisturbed clay and the filter-paper drain. In the real soil with a smeared layer and a drain both of finite permeability there will be a general vertical component of flow in the specimen even in the absence of end drainage, so that, strictly speaking, for

[3] R. F. Scott, "A New Method of Consolidation Coefficient Evaluation," *Journal of the Soil Mechanics and Foundation Division*, Am. Soc. Civil Engrs., Vol. 87, SM1, February, 1961, and closure, Vol. 88, SM1, February, 1962.

[4] R. A. Barron, "Consolidation of Fine-Grained Soils By Drain Wells," *Transactions*, Am. Soc. Civil Engrs., Vol. 113, 1948, p. 718.

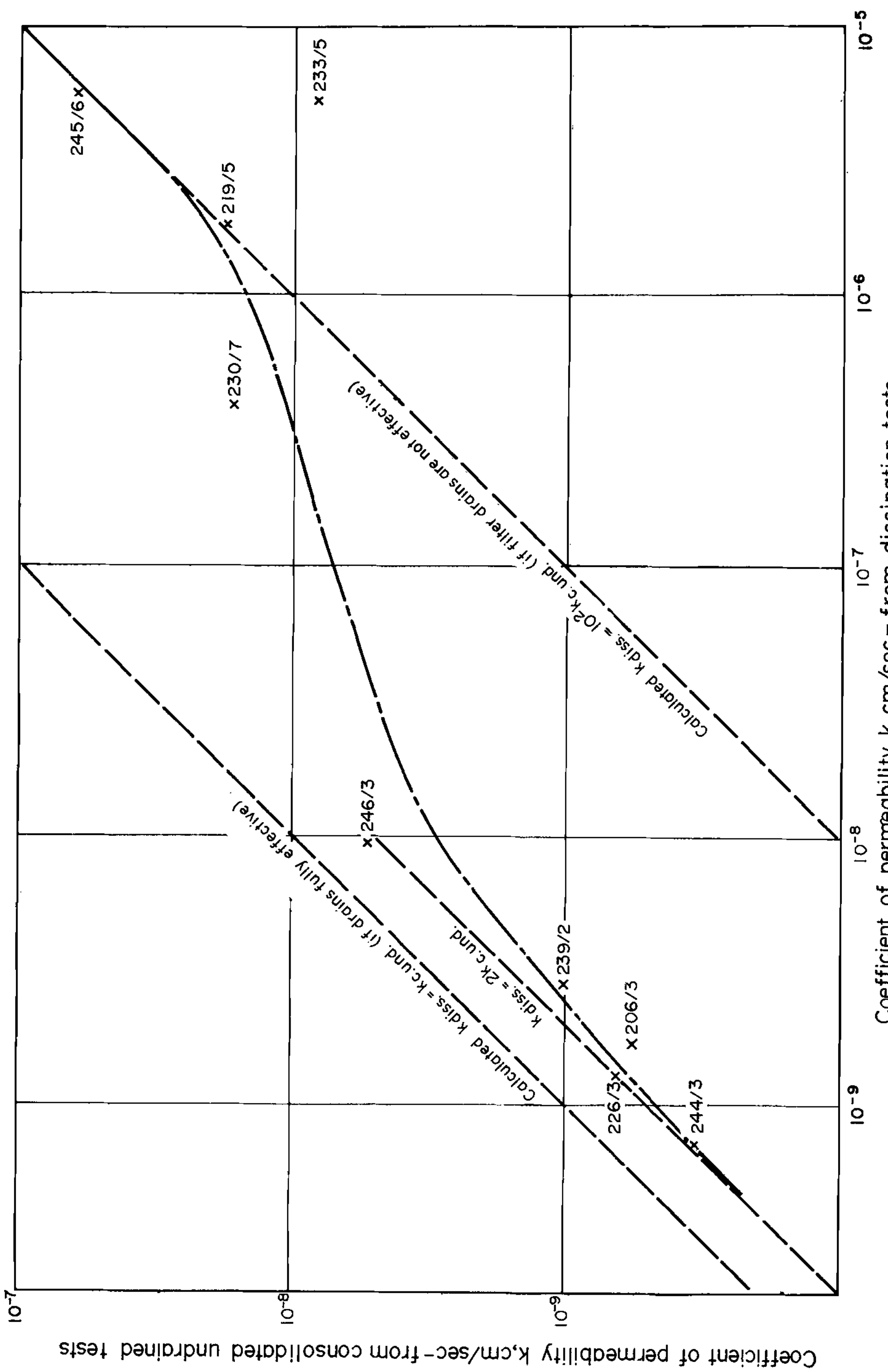

FIG. 10—Coefficient of Permeability From Dissipation and Consolidated-Undrained Tests.

the equal strain problem, a solution should be obtained to Eq (30) in which the right-hand side is a function of time, independent of r. This was not done, however, because of the extra difficulty involved and the smaller importance of the vertical flow contribution in a stratified soil.

The equal strain solution was obtained for radial flow only in the specimen by assuming $k_z = 0$. Thus, although the excess pore pressure obtained in the solution was a function of z as well as the other variables, no flow derived from the vertical pressure gradients.

Thus in the specimen, the excess pore pressure is given by

$$\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = f(z, t) \ldots\ldots (49)$$

The initial condition throughout the specimen is

$$u(r,z,0) = \bar{u}_0 \ldots\ldots\ldots\ldots (50)$$

and the equal-strain condition to be satisfied is that the rate of flow of water across any radius r is proportional to the rate of compression of the area within r, or

$$-\frac{2k_r}{\gamma_w}\frac{\partial u(r, z, t)}{\partial r} = \frac{r}{1+e}\frac{\partial e}{\partial t} \ldots\ldots (51)$$

The boundary condition is

$$\frac{\partial u(0, z, t)}{\partial r} = 0 \ldots\ldots\ldots\ldots (52)$$

in addition to Eq (48).

The side drain is assumed to be of small thickness and incompressible so that the excess pore pressure $v(z, t)$ is uniform through the drain and the following equation therefore holds for the drain.

$$hk_p \frac{\partial^2 v}{\partial z^2} + K(u - v) = 0 \ldots\ldots (53)$$

The boundary conditions for the drain are

$$v(d,t) = 0 \ldots\ldots\ldots\ldots (54)$$

$$\frac{\partial v(o, t)}{\partial z} = 0 \ldots\ldots\ldots\ldots (55)$$

Under these conditions the solutions for the pore pressures u and v in the soil and in the drain respectively are

$$u(r, z, t) = \bar{u}_0 \frac{\cosh \beta z}{\cosh \beta d}\left\{\frac{1}{m}\left[1 + m - \frac{2r^2}{a^2}\right] + \left(1 - \frac{\cosh \beta z}{\cosh \beta d}\right)\right\} \cdot \exp\left(-\frac{8T_r}{m}\frac{\cosh \beta z}{\cosh \beta d}\right) \ldots (56)$$

$$v(z, t) = \bar{u}_0\left(1 - \frac{\cosh \beta z}{\cosh \beta d}\right) \cdot \exp\left(-\frac{8T_r}{m}\frac{\cosh \beta z}{\cosh \beta d}\right) \ldots (57)$$

where

$$\beta = \frac{\pi}{d\nu}; \quad m = 1 + \frac{4k_r}{Ka}; \quad T_r = \frac{c_{ar}t}{a^2}.$$

The parameter β is related to the authors' ν which describes the effectiveness of the filter drain, whereas m is indicative of the amount of smearing which has taken place at the specimen periphery. When the drains have infinite permeability $\beta = 0$ ($\nu = \infty$) and when there is no surface resistance due to smear, $m = 1$.

The purpose of the graph in Fig. 10 (opposite page) is to show the effect of using filter drains on the calculation of the permeability.

The results from consolidated-undrained and dissipation tests on nine specimens have been plotted. Corresponding values of permeability have been given. The compressibility of the soil and the effective stress ranges have been studied so that the permeability values plotted are reasonably representative of the range of the tests. The results of the dissipation and consolidated-undrained tests do not, however, agree exactly in these respects.

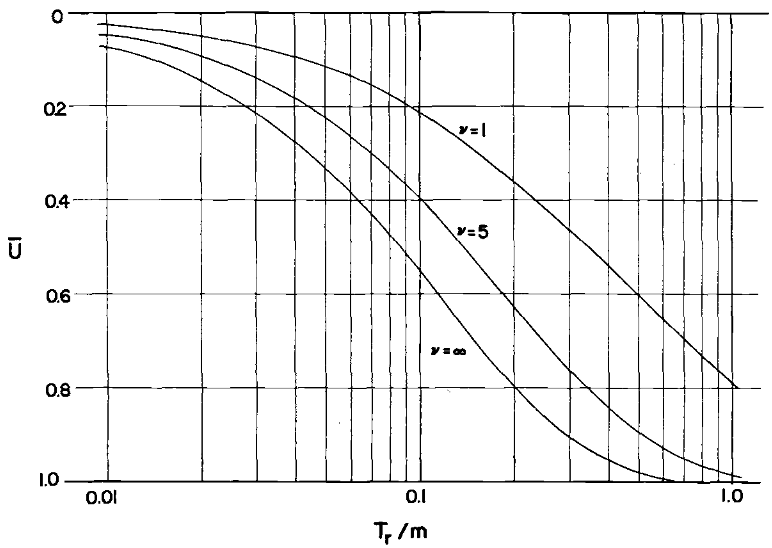

FIG. 11—Average Degree of Consolidation Versus Time for Equal Strain Solution, Radial Flow Only.

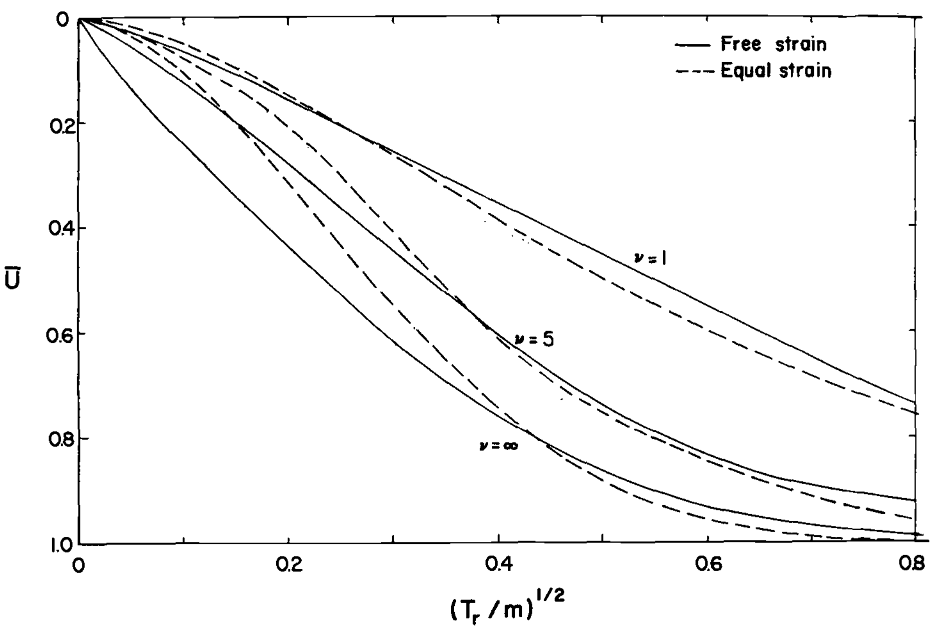

FIG. 12—Comparison of Free Strain and Equal Strain Solutions for Combined Radial and Vertical Flow.

The average pore pressure $\bar{u}(z, t)$ in the specimen at any level z is given by the expression

$$\bar{u}(z, t) = \bar{u}_0 \exp\left(-\frac{8T_r}{m}\frac{\cosh \beta z}{\cosh \beta d}\right) \quad (58)$$

and the over-all average degree of consolidation is

$$\bar{U} = 1 - \frac{1}{d}\int_0^d \exp\left(-\frac{8T_r}{m}\frac{\cosh \beta z}{\cosh \beta d}\right)dz \quad (59)$$

which then corresponds to the authors' Eq (42) for the different case studied.

In the case that $\beta = 0$ (infinitely permeable drain), Eq (59) reduces to

$$\bar{U} = 1 - e^{-8T/m} \quad (60)$$

which the writer derived and plotted previously.[5] Since the factor m is dimensionless, it can be employed with T_r to give a modified time factor, representing the slowing of the drainage process by the smeared layer.

Equation (59) has been integrated numerically for the cases $\nu = 1$ and $\nu = 5$ and Eq (60) evaluated to give the results plotted on Fig. 11. It is emphasized that these curves represent the case of radial drainage only under the conditions studied.

In order to take into account vertical flow to porous capping stones it is necessary to combine the result of evaluating Eq (59) with the solution for vertical drainage only in the specimen, with either finite or infinite permeability in the porous plate, as the authors have considered. The combination can be effected by the method of Newman[6] where

$$\bar{U}(r,z) = 1 - [1 - \bar{U}(r)][1 - U(z)] \quad (61)$$

When the dimensions, drainage conditions, and ratio of vertical to lateral permeability of the specimen are known, the vertical time factor can be related to the radial time factor, T_r, and the results of the application of Eq (61) represented in terms of T_r.

In one case studied by the authors, the ratio $a\sqrt{k_z}/d\sqrt{k_r} = \frac{1}{4}$, and the results are plotted in Fig. 8. Using the one-dimensional consolidation solution, $\nu = 0$, the results of Fig. 11, and Eq (61), the writer has computed the corresponding curves for the equal vertical strain case, and these are presented in Fig. 12 along with the authors' curves for comparison. The equal strain curves are correctly shown as crossing the free strain curves.

It is seen from Fig. 12 that the geometrical requirement of equal strain causes the resulting consolidation curves to deviate from straight lines on the square root of time plot, and that straight lines tangential to these curves at their inflection points would have slopes and intercepts considerably at variance with those found from the corresponding free strain curves. It is possible that a more general fitting method[3] whose functional curves can be plotted from the mathematical results of the present paper or discussions would prove more suitable for the analysis of coefficient of consolidation than the square root of time method.

However, it is not, perhaps, too obvious to mention that a knowledge of the processes occurring in and constraints applied to a specimen is required for selection of the appropriate mathematical solution with which the real material behavior is to be identified; otherwise, numerical data will be obtained which are only distantly related, or even unrelated, to physical parameters of concern.

A. W. BISHOP AND R. E. GIBSON (*authors' closure*)—The authors are in-

[5] R. F. Scott, *Principles of Soil Mechanics*, Addison-Wesley, Reading, 1963.

[6] A. B. Newman, "The Drying of Porous Solids," *Transactions*, American Institute of Chemical Engineers, Vol. 27, 1931, p. 310.

debted to Dr. Scott for presenting a most illuminating analysis of the triaxial consolidation test based on the assumption of equal vertical strain. The importance of whether the free-strain or equal-strain assumption is made is well illustrated in Fig. 12. The fact that intuitive assumptions of this kind have to be invoked at all emphasises the rather unsatisfactory nature of unlinked theories of three-dimensional consolidation of the Terzaghi type. Unfortunately, the only completely linked theory, due to M. A. Biot, is based on the assumption of a perfectly elastic soil skeleton, but it would appear that the task of solving a problem of the complexity considered, with this particular restriction eased, would be extremely formidable.

The interesting test data presented by Mr. Corbett confirms broadly the thesis advanced by the authors. Clearly, a much more detailed investigation would be required to decide between the hypotheses of free strain and equal strain, but the further clarification that might be obtained would probably not justify the experimental sophistication required.