APPENDIX

TABLES OF SQUARES AND SQUARE ROOTS

SQUARES AND SQUARE ROOTS 1–200

No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root
1	1	1.0000	51	2 601	7.1414	101	10 201	10.0499	151	22 801	12.2882
2	4	1.4142	52	2 704	7.2111	102	10 404	10.0995	152	23 104	12.3288
3	9	1.7321	53	2 809	7.2801	103	10 609	10.1489	153	23 409	12.3693
4	16	2.0000	54	2 916	7.3485	104	10 816	10.1980	154	23 716	12.4097
5	25	2.2361	55	3 025	7.4162	105	11 025	10.2470	155	24 025	12.4499
6	36	2.4495	56	3 136	7.4833	106	11 236	10.2956	156	24 336	12.4900
7	49	2.6458	57	3 249	7.5498	107	11 449	10.3441	157	24 649	12.5300
8	64	2.8284	58	3 364	7.6158	108	11 664	10.3923	158	24 964	12.5698
9	81	3.0000	59	3 481	7.6811	109	11 881	10.4403	159	25 281	12.6095
10	100	3.1623	60	3 600	7.7460	110	12 100	10.4881	160	25 600	12.6491
11	121	3.3166	61	3 721	7.8102	111	12 321	10.5357	161	25 921	12 6886
12	144	3.4641	62	3 844	7.8740	112	12 544	10.5830	162	26 244	12.7279
13	169	3.6056	63	3 969	7.9373	113	12 769	10.6301	163	26 569	12.7671
14	196	3.7417	64	4 096	8.0000	114	12 996	10.6771	164	26 896	12.8062
15	225	3.8730	65	4 225	8.0623	115	13 225	10.7238	165	27 225	12.8452
16	256	4.0000	66	4 356	8.1240	116	13 456	10.7703	166	27 556	12 8841
17	289	4.1231	67	4 489	8.1854	117	13 689	10.8167	167	27 889	12.9228
18	324	4.2426	68	4 624	8.2462	118	13 924	10.8628	168	28 224	12.9615
19	361	4.3589	69	4 761	8.3066	119	14 161	10.9087	169	28 561	13.0000
20	400	4.4721	70	4 900	8.3666	120	14 400	10.9545	170	28 900	13.0384
21	441	4.5826	71	5 041	8.4261	121	14 641	11.0000	171	29 241	13.0767
22	484	4.6904	72	5 184	8.4853	122	14 884	11.0454	172	29 584	13.1149
23	529	4.7958	73	5 329	8.5440	123	15 129	11.0905	173	29 929	13.1529
24	576	4.8990	74	5 476	8.6023	124	15 376	11.1355	174	30 276	13.1909
25	625	5.0000	75	5 625	8.6603	125	15 625	11.1803	175	30 625	13.2288
26	676	5.0990	76	5 776	8.7178	126	15 876	11.2250	176	30 976	13 2665
27	729	5.1962	77	5 929	8.7750	127	16 129	11.2694	177	31 329	13.3041
28	784	5.2915	78	6 084	8.8318	128	16 384	11.3137	178	31 684	13.3417
29	841	5.3852	79	6 241	8.8882	129	16 641	11.3578	179	32 041	13.3791
30	900	5.4772	80	6 400	8.9443	130	16 900	11.4018	180	32 400	13.4164
31	961	5.5678	81	6 561	9.0000	131	17 161	11.4455	181	32 761	13.4536
32	1 024	5.6569	82	6 724	9.0554	132	17 424	11.4891	182	33 124	13.4907
33	1 089	5.7446	83	6 889	9.1104	133	17 689	11.5326	183	33 489	13.5277
34	1 156	5.8310	84	7 056	9.1652	134	17 956	11.5758	184	33 856	13.5647
35	1 225	5.9161	85	7 225	9.2195	135	18 225	11.6190	185	34 225	13.6015
36	1 296	6.0000	86	7 396	9.2736	136	18 496	11.6619	186	34 596	13.6382
37	1 369	6.0828	87	7 569	9.3274	137	18 769	11.7047	187	34 969	13.6748
38	1 444	6.1644	88	7 744	9.3808	138	19 044	11.7473	188	35 344	13.7113
39	1 521	6.2450	89	7 921	9.4340	139	19 321	11.7898	189	35 721	13.7477
40	1 600	6.3246	90	8 100	9.4868	140	19 600	11.8322	190	36 100	13.7840
41	1 681	6.4031	91	8 281	9.5394	141	19 881	11.8743	191	36 481	13.8203
42	1 764	6.4807	92	8 464	9.5917	142	20 164	11.9164	192	36 864	13.8564
43	1 849	6.5574	93	8 649	9.6437	143	20 449	11.9583	193	37 249	13.8924
44	1 936	6.6332	94	8 836	9.6954	144	20 736	12.0000	194	37 636	13.9284
45	2 025	6.7082	95	9 025	9.7468	145	21 025	12.0416	195	38 025	13.9642
46	2 116	6.7823	96	9 216	9.7980	146	21 316	12.0830	196	38 416	14.0000
47	2 209	6.8557	97	9 409	9.8489	147	21 609	12.1244	197	38 809	14.0357
48	2 304	6.9282	98	9 604	9.8995	148	21 904	12.1655	198	39 204	14.0712
49	2 401	7.0000	99	9 801	9.9499	149	22 201	12.2066	199	39 601	14.1067
50	2 500	7.0711	100	10 000	10.0000	150	22 500	12.2474	200	40 000	14.1421

NOTE.—To find the square root of numbers greater than 2000, look up the given number in the "Square" column and obtain the answer from the corresponding value in the "No." column

Example 1.—To find the square root of 2174.386.

Group the digits in pairs starting from the decimal point, thus:

$$\widehat{21}\ \widehat{74}.\ \widehat{38}\ \widehat{60}$$

There will always be one digit in the square root for each group in the given number. Observe that the square root of the number in the first group (21) is between 4 and 5. Referring to the table, find in the "No." column the two numbers lying between 400 and 500 whose squares most nearly equal the given number. In this case the numbers are 466 and 467. Interpolating and locating the decimal point gives 46.63 for the desired square root.

In referring to the table, numbers between 40 and 50 in the "No." column may be used as well as those between 400 and 500, but the latter numbers will give the desired square root to one more significant figure.

Example 2.—To find the square root of 21743.86.

Group the digits in pairs as before:

$$\widehat{2}\ \widehat{17}\ \widehat{43}.\ \widehat{86}$$

and observe that the square root of the number in the first group (2) is between 1 and 2. Refer to the "No." column of the table for numbers lying between 1000 and 2000. In this case, the numbers are 1474 and 1475, and interpolating and locating the decimal point gives 147.46.

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201–500 SQUARES AND SQUARE ROOTS (*Continued*)

No	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root
201	40 401	14.1774	276	76 176	16.6132	351	123 201	18.7350	426	181 476	20.6398
202	40 804	14.2127	277	76 729	16.6433	352	123 904	18.7617	427	182 329	20.6640
203	41 209	14.2478	278	77 284	16.6733	353	124 609	18.7883	428	183 184	20.6882
204	41 616	14.2829	279	77 841	16.7033	354	125 316	18.8149	429	184 041	20.7123
205	42 025	14.3178	280	78 400	16.7332	355	126 025	18.8414	430	184 900	20.7364
206	42 436	14.3527	281	78 961	16.7631	356	126 736	18.8680	431	185 761	20.7605
207	42 849	14.3875	282	79 524	16.7929	357	127 449	18.8944	432	186 624	20.7846
208	43 264	14.4222	283	80 089	16.8226	358	128 164	18.9209	433	187 489	20.8087
209	43 681	14.4568	284	80 656	16.8523	359	128 881	18.9473	434	188 356	20.8327
210	44 100	14.4914	285	81 225	16.8819	360	129 600	18.9737	435	189 225	20.8567
211	44 521	14.5258	286	81 796	16.9115	361	130 321	19.0000	436	190 096	20.8806
212	44 944	14.5602	287	82 369	16.9411	362	131 044	19.0263	437	190 969	20.9045
213	45 369	14.5945	288	82 944	16.9706	363	131 769	19.0526	438	191 844	20.9284
214	45 796	14.6287	289	83 521	17.0000	364	132 496	19.0788	439	192 721	20.9523
215	46 225	14.6629	290	84 100	17.0294	365	133 225	19.1050	440	193 600	20.9762
216	46 656	14.6969	291	84 681	17.0587	366	133 956	19.1311	441	194 481	21.0000
217	47 089	14.7309	292	85 264	17.0880	367	134 689	19.1572	442	195 364	21.0238
218	47 524	14.7648	293	85 849	17.1172	368	135 424	19.1833	443	196 249	21.0476
219	47 961	14.7986	294	86 436	17.1464	369	136 161	19.2094	444	197 136	21.0713
220	48 400	14.8324	295	87 025	17.1756	370	136 900	19.2354	445	198 025	21.0950
221	48 841	14.8661	296	87 616	17.2047	371	137 641	19.2614	446	198 916	21.1187
222	49 284	14.8997	297	88 209	17.2337	372	138 384	19.2873	447	199 809	21.1424
223	49 729	14.9332	298	88 804	17.2627	373	139 129	19.3132	448	200 704	21.1660
224	50 176	14.9666	299	89 401	17.2916	374	139 876	19.3391	449	201 601	21.1896
225	50 625	15.0000	300	90 000	17.3205	375	140 625	19.3649	450	202 500	21.2132
226	51 076	15.0333	301	90 601	17.3494	376	141 376	19.3907	451	203 401	21.2368
227	51 529	15.0665	302	91 204	17.3781	377	142 129	19.4165	452	204 304	21.2603
228	51 984	15.0997	303	91 809	17.4069	378	142 884	19.4422	453	205 209	21.2838
229	52 441	15.1327	304	92 416	17.4356	379	143 641	19.4679	454	206 116	21.3073
230	52 900	15.1658	305	93 025	17.4642	380	144 400	19.4936	455	207 025	21.3307
231	53 361	15.1987	306	93 636	17.4929	381	145 161	19.5192	456	207 936	21.3542
232	53 824	15.2315	307	94 249	17.5214	382	145 924	19.5448	457	208 849	21.3776
233	54 289	15.2643	308	94 864	17.5499	383	146 689	19.5704	458	209 764	21.4009
234	54 756	15.2971	309	95 481	17.5784	384	147 456	19.5959	459	210 681	21.4243
235	55 225	15.3297	310	96 100	17.6068	385	148 225	19.6214	460	211 600	21.4476
236	55 696	15.3623	311	96 721	17.6352	386	148 996	19.6469	461	212 521	21.4709
237	56 169	15.3948	312	97 344	17.6635	387	149 769	19.6723	462	213 444	21.4942
238	56 644	15.4272	313	97 969	17.6918	388	150 544	19.6977	463	214 369	21.5174
239	57 121	15.4596	314	98 596	17.7200	389	151 321	19.7231	464	215 296	21.5407
240	57 600	15.4919	315	99 225	17.7482	390	152 100	19.7484	465	216 225	21.5639
241	58 081	15.5242	316	99 856	17.7764	391	152 881	19.7737	466	217 156	21.5870
242	58 564	15.5563	317	100 489	17.8045	392	153 664	19.7990	467	218 089	21.6102
243	59 049	15.5885	318	101 124	17.8326	393	154 449	19.8242	468	219 024	21.6333
244	59 536	15.6205	319	101 761	17.8606	394	155 236	19.8494	469	219 961	21.6564
245	60 025	15.6525	320	102 400	17.8885	395	156 025	19.8746	470	220 900	21.6795
246	60 516	15.6844	321	103 041	17.9165	396	156 816	19.8997	471	221 841	21.7025
247	61 009	15.7162	322	103 684	17.9444	397	157 609	19.9249	472	222 784	21.7256
248	61 504	15.7480	323	104 329	17.9722	398	158 404	19.9499	473	223 729	21.7486
249	62 001	15.7797	324	104 976	18.0000	399	159 201	19.9750	474	224 676	21.7715
250	62 500	15.8114	325	105 625	18.0278	400	160 000	20.0000	475	225 625	21.7945
251	63 001	15.8430	326	106 276	18.0555	401	160 801	20.0250	476	226 576	21.8174
252	63 504	15.8745	327	106 929	18.0831	402	161 604	20.0499	477	227 529	21.8403
253	64 009	15.9060	328	107 584	18.1108	403	162 409	20.0749	478	228 484	21.8632
254	64 516	15.9374	329	108 241	18.1384	404	163 216	20.0998	479	229 441	21.8861
255	65 025	15.9687	330	108 900	18.1659	405	164 025	20.1246	480	230 400	21.9089
256	65 536	16.0000	331	109 561	18.1934	406	164 836	20.1494	481	231 361	21.9317
257	66 049	16.0312	332	110 224	18.2209	407	165 649	20.1742	482	232 324	21.9545
258	66 564	16.0624	333	110 889	18.2483	408	166 464	20.1990	483	233 289	21.9773
259	67 081	16.0935	334	111 556	18.2757	409	167 281	20.2237	484	234 256	22.0000
260	67 600	16.1245	335	112 225	18.3030	410	168 100	20.2485	485	235 225	22.0227
261	68 121	16.1555	336	112 896	18.3303	411	168 921	20.2731	486	236 196	22.0454
262	68 644	16.1864	337	113 569	18.3576	412	169 744	20.2978	487	237 169	22.0681
263	69 169	16.2173	338	114 244	18.3848	413	170 569	20.3224	488	238 144	22.0907
264	69 696	16.2481	339	114 921	18.4120	414	171 396	20.3470	489	239 121	22.1133
265	70 225	16.2788	340	115 600	18.4391	415	172 225	20.3715	490	240 100	22.1359
266	70 756	16.3095	341	116 281	18.4662	416	173 056	20.3961	491	241 081	22.1585
267	71 289	16.3401	342	116 964	18.4932	417	173 889	20.4206	492	242 064	22.1811
268	71 824	16.3707	343	117 649	18.5203	418	174 724	20.4450	493	243 049	22.2036
269	72 361	16.4012	344	118 336	18.5472	419	175 561	20.4695	494	244 036	22.2261
270	72 900	16.4317	345	119 025	18.5742	420	176 400	20.4939	495	245 025	22.2486
271	73 441	16.4621	346	119 716	18.6011	421	177 241	20.5183	496	246 016	22.2711
272	73 984	16.4924	347	120 409	18.6279	422	178 084	20.5426	497	247 009	22.2935
273	74 529	16.5227	348	121 104	18.6548	423	178 929	20.5670	498	248 004	22.3159
274	75 076	16.5529	349	121 801	18.6815	424	179 776	20.5913	499	249 001	22.3383
275	75 625	16.5831	350	122 500	18.7083	425	180 625	20.6155	500	250 000	22.3607

SQUARES AND SQUARE ROOTS (*Continued*) 501–800

No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root
501	251 001	22.3830	576	331 776	24.0000	651	423 801	25.5147	726	527 076	26.9444
502	252 004	22.4054	577	332 929	24.0208	652	425 104	25.5343	727	528 529	26.9629
503	253 009	22.4277	578	334 084	24.0416	653	426 409	25.5539	728	529 984	26.9815
504	254 016	22.4499	579	335 241	24.0624	654	427 716	25.5734	729	531 441	27.0000
505	255 025	22.4722	580	336 400	24.0832	655	429 025	25.5930	730	532 900	27.0185
506	256 036	22.4944	581	337 561	24.1039	656	430 336	25.6125	731	534 361	27.0370
507	257 049	22.5167	582	338 724	24.1247	657	431 649	25.6320	732	535 824	27.0555
508	258 064	22.5389	583	339 889	24.1454	658	432 964	25.6515	733	537 289	27.0740
509	259 081	22.5610	584	341 056	24.1661	659	434 281	25.6710	734	538 756	27.0924
510	260 100	22.5832	585	342 225	24.1868	660	435 600	25.6905	735	540 225	27.1109
511	261 121	22.6053	586	343 396	24.2074	661	436 921	25.7099	736	541 696	27.1293
512	262 144	22.6274	587	344 569	24.2281	662	438 244	25.7294	737	543 169	27.1477
513	263 169	22.6495	588	345 744	24.2487	663	439 569	25.7488	738	544 644	27.1662
514	264 196	22.6716	589	346 921	24.2693	664	440 896	25.7682	739	546 121	27.1846
515	265 225	22.6936	590	348 100	24.2899	665	442 225	25.7876	740	547 600	27.2029
516	266 256	22.7156	591	349 281	24.3105	666	443 556	25.8070	741	549 081	27.2213
517	267 289	22.7376	592	350 464	24.3311	667	444 889	25.8263	742	550 564	27.2397
518	268 324	22.7596	593	351 649	24.3516	668	446 224	25.8457	743	552 049	27.2580
519	269 361	22.7816	594	352 836	24.3721	669	447 561	25.8650	744	553 536	27.2764
520	270 400	22.8035	595	354 025	24.3926	670	448 900	25.8844	745	555 025	27.2947
521	271 441	22.8254	596	355 216	24.4131	671	450 241	25.9037	746	556 516	27.3130
522	272 484	22.8473	597	356 409	24.4336	672	451 584	25.9230	747	558 009	27.3313
523	273 529	22.8692	598	357 604	24.4540	673	452 929	25.9422	748	559 504	27.3496
524	274 576	22.8910	599	358 801	24.4745	674	454 276	25.9615	749	561 001	27.3679
525	275 625	22.9129	600	360 000	24.4949	675	455 625	25.9808	750	562 500	27.3861
526	276 676	22.9347	601	361 201	24.5153	676	456 976	26.0000	751	564 001	27.4044
527	277 729	22.9565	602	362 404	24.5357	677	458 329	26.0192	752	565 504	27.4226
528	278 784	22.9783	603	363 609	24.5561	678	459 684	26.0384	753	567 009	27.4408
529	279 841	23.0000	604	364 816	24.5764	679	461 041	26.0576	754	568 516	27.4591
530	280 900	23.0217	605	366 025	24.5967	680	462 400	26.0768	755	570 025	27.4773
531	281 961	23.0434	606	367 236	24.6171	681	463 761	26.0960	756	571 536	27.4955
532	283 024	23.0651	607	368 449	24.6374	682	465 124	26.1151	757	573 049	27.5136
533	284 089	23.0868	608	369 664	24.6577	683	466 489	26.1343	758	574 564	27.5318
534	285 156	23.1084	609	370 881	24.6779	684	467 856	26.1534	759	576 081	27.5500
535	286 225	23.1301	610	372 100	24.6982	685	469 225	26.1725	760	577 600	27.5681
536	287 296	23.1517	611	373 321	24.7184	686	470 596	26.1916	761	579 121	27.5862
537	288 369	23.1733	612	374 544	24.7386	687	471 969	26.2107	762	580 644	27.6043
538	289 444	23.1948	613	375 769	24.7588	688	473 344	26.2298	763	582 169	27.6225
539	290 521	23.2164	614	376 996	24.7790	689	474 721	26.2488	764	583 696	27.6405
540	291 600	23.2379	615	378 225	24.7992	690	476 100	26.2679	765	585 225	27.6586
541	292 681	23.2594	616	379 456	24.8193	691	477 481	26.2869	766	586 756	27.6767
542	293 764	23.2809	617	380 689	24.8395	692	478 864	26.3059	767	588 289	27.6948
543	294 849	23.3024	618	381 924	24.8596	693	480 249	26.3249	768	589 824	27.7128
544	295 936	23.3238	619	383 161	24.8797	694	481 636	26.3439	769	591 361	27.7308
545	297 025	23.3452	620	384 400	24.8998	695	483 025	26.3629	770	592 900	27.7489
546	298 116	23.3666	621	385 641	24.9199	696	484 416	26.3818	771	594 441	27.7669
547	299 209	23.3880	622	386 884	24.9399	697	485 809	26.4008	772	595 984	27.7849
548	300 304	23.4094	623	388 129	24.9600	698	487 204	26.4197	773	597 529	27.8029
549	301 401	23.4307	624	389 376	24.9800	699	488 601	26.4386	774	599 076	27.8209
550	302 500	23.4521	625	390 625	25.0000	700	490 000	26.4575	775	600 625	27.8388
551	303 601	23.4734	626	391 876	25.0200	701	491 401	26.4764	776	602 176	27.8568
552	304 704	23.4947	627	393 129	25.0400	702	492 804	26.4953	777	603 729	27.8747
553	305 809	23.5160	628	394 384	25.0599	703	494 209	26.5141	778	605 284	27.8927
554	306 916	23.5372	629	395 641	25.0799	704	495 616	26.5330	779	606 841	27.9106
555	308 025	23.5584	630	396 900	25.0998	705	497 025	26.5518	780	608 400	27.9285
556	309 136	23.5797	631	398 161	25.1197	706	498 436	26.5707	781	609 961	27.9464
557	310 249	23.6008	632	399 424	25.1396	707	499 849	26.5895	782	611 524	27.9643
558	311 364	23.6220	633	400 689	25.1595	708	501 264	26.6083	783	613 089	27.9821
559	312 481	23.6432	634	401 956	25.1794	709	502 681	26.6271	784	614 656	28.0000
560	313 600	23.6643	635	403 225	25.1992	710	504 100	26.6458	785	616 225	28.0179
561	314 721	23.6854	636	404 496	25.2190	711	505 521	26.6646	786	617 796	28 0357
562	315 844	23.7065	637	405 769	25.2389	712	506 944	26.6833	787	619 369	28.0535
563	316 969	23.7276	638	407 044	25.2587	713	508 369	26.7021	788	620 944	28.0713
564	318 096	23.7487	639	408 321	25.2784	714	509 796	26.7208	789	622 521	28.0891
565	319 225	23.7697	640	409 600	25.2982	715	511 225	26.7395	790	624 100	28.1069
566	320 356	23.7908	641	410 881	25.3180	716	512 656	26.7582	791	625 681	28.1247
567	321 489	23.8118	642	412 164	25.3377	717	514 089	26.7769	792	627 264	28.1425
568	322 624	23.8328	643	413 449	25.3574	718	515 524	26.7955	793	628 849	28.1603
569	323 761	23.8537	644	414 736	25.3772	719	516 961	26.8142	794	630 436	28.1780
570	324 900	23.8747	645	416 025	25.3969	720	518 400	26.8328	795	632 025	28.1957
571	326 041	23.8956	646	417 316	25.4165	721	519 841	26.8514	796	633 616	28.2135
572	327 184	23.9165	647	418 609	25.4362	722	521 284	26.8701	797	635 209	28.2312
573	328 329	23.9374	648	419 904	25.4558	723	522 729	26.8887	798	636 804	28.2489
574	329 476	23.9583	649	421 201	25.4755	724	524 176	26.9072	799	638 401	28.2666
575	330 625	23.9792	650	422 500	25.4951	725	525 625	26.9258	800	640 000	28.2843

801–1100 SQUARES AND SQUARE ROOTS (*Continued*)

No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root
801	641 601	28.3019	876	767 376	29.5973	951	904 401	30.8383	1026	1 052 676	32.0312
802	643 204	28.3196	877	769 129	29.6142	952	906 304	30.8545	1027	1 054 729	32.0468
803	644 809	28.3373	878	770 884	29.6311	953	908 209	30.8707	1028	1 056 784	32.0624
804	646 416	28.3549	879	772 641	29.6479	954	910 116	30.8869	1029	1 058 841	32.0780
805	648 025	28.3725	880	774 400	29.6648	955	912 025	30.9031	1030	1 060 900	32.0936
806	649 636	28.3901	881	776 161	29.6816	956	913 936	30.9192	1031	1 062 961	32.1092
807	651 249	28.4077	882	777 924	29.6985	957	915 849	30.9354	1032	1 065 024	32.1248
808	652 864	28.4253	883	779 689	29.7153	958	917 764	30.9516	1033	1 067 089	32.1403
809	654 481	28.4429	884	781 456	29.7321	959	919 681	30.9677	1034	1 069 156	32.1559
810	656 100	28.4605	885	783 225	29.7489	960	921 600	30.9839	1035	1 071 225	32.1714
811	657 721	28.4781	886	784 996	29.7658	961	923 521	31.0000	1036	1 073 296	32.1870
812	659 344	28.4956	887	786 769	29.7825	962	925 444	31.0161	1037	1 075 369	32.2025
813	660 969	28.5132	888	788 544	29.7993	963	927 369	31.0322	1038	1 077 444	32.2180
814	662 596	28.5307	889	790 321	29.8161	964	929 296	31.0483	1039	1 079 521	32.2335
815	664 225	28.5482	890	792 100	29.8329	965	931 225	31.0644	1040	1 081 600	32.2490
816	665 856	28.5657	891	793 881	29.8496	966	933 156	31.0805	1041	1 083 681	32.2645
817	667 489	28.5832	892	795 664	29.8664	967	935 089	31.0966	1042	1 085 764	32.2800
818	669 124	28.6007	893	797 449	29.8831	968	937 024	31.1127	1043	1 087 849	32.2955
819	670 761	28.6182	894	799 236	29.8998	969	938 961	31.1288	1044	1 089 936	32.3110
820	672 400	28.6356	895	801 025	29.9166	970	940 900	31.1448	1045	1 092 025	32.3265
821	674 041	28.6531	896	802 816	29.9333	971	942 841	31.1609	1046	1 094 116	32 3419
822	675 684	28.6705	897	804 609	29.9500	972	944 784	31.1769	1047	1 096 209	32.3574
823	677 329	28.6880	898	806 404	29.9666	973	946 729	31.1929	1048	1 098 304	32.3728
824	678 976	28.7054	899	808 201	29.9833	974	948 676	31.2090	1049	1 100 401	32.3883
825	680 625	28.7228	900	810 000	30.0000	975	950 625	31.2250	1050	1 102 500	32.4037
826	682 276	28.7402	901	811 801	30.0167	976	952 576	31.2410	1051	1 104 601	32.4191
827	683 929	28.7576	902	813 604	30.0333	977	954 529	31.2570	1052	1 106 704	32.4345
828	685 584	28.7750	903	815 409	30.0500	978	956 484	31.2730	1053	1 108 809	32.4500
829	687 241	28.7924	904	817 216	30.0666	979	958 441	31.2890	1054	1 110 916	32.4654
830	688 900	28.8097	905	819 025	30.0832	980	960 400	31.3050	1055	1 113 025	32.4808
831	690 561	28.8271	906	820 836	30.0998	981	962 361	31.3209	1056	1 115 136	32.4962
832	692 224	28 8444	907	822 649	30.1164	982	964 324	31.3369	1057	1 117 249	32.5115
833	693 889	28 8617	908	824 464	30.1330	983	966 289	31.3528	1058	1 119 364	32.5269
834	695 556	28 8791	909	826 281	30.1496	984	968 256	31.3688	1059	1 121 481	32.5423
835	697 225	28.8964	910	828 100	30.1662	985	970 225	31.3847	1060	1 123 600	32.5576
836	698 896	28.9137	911	829 921	30.1828	986	972 196	31.4006	1061	1 125 721	32.5730
837	700 569	28.9310	912	831 744	30.1993	987	974 169	31.4166	1062	1 127 844	32.5883
838	702 244	28.9482	913	833 569	30.2159	988	976 144	31.4325	1063	1 129 969	32.6037
839	703 921	28.9655	914	835 396	30.2324	989	978 121	31.4484	1064	1 132 096	32.6190
840	705 600	28.9828	915	837 225	30.2490	990	980 100	31.4643	1065	1 134 225	32.6343
841	707 281	29.0000	916	839 056	30.2655	991	982 081	31.4802	1066	1 136 356	32.6497
842	708 964	29.0172	917	840 889	30.2820	992	984 064	31.4960	1067	1 138 489	32 6650
843	710 649	29.0345	918	842 724	30.2985	993	986 049	31.5119	1068	1 140 624	32.6803
844	712 336	29.0517	919	844 561	30.3150	994	988 036	31.5278	1069	1 142 761	32.6956
845	714 025	29.0689	920	846 400	30.3315	995	990 025	31.5436	1070	1 144 900	32.7109
846	715 716	29.0861	921	848 241	30.3480	996	992 016	31.5595	1071	1 147 041	32.7261
847	717 409	29.1033	922	850 084	30.3645	997	994 009	31.5753	1072	1 149 184	32.7414
848	719 104	29.1204	923	851 929	30.3809	998	996 004	31.5911	1073	1 151 329	32.7567
849	720 801	29.1376	924	853 776	30.3974	999	998 001	31.6070	1074	1 153 476	32.7719
850	722 500	29.1548	925	855 625	30.4138	1000	1 000 000	31.6228	1075	1 155 625	32.7872
851	724 201	29.1719	926	857 476	30.4302	1001	1 002 001	31.6386	1076	1 157 776	32.8024
852	725 904	29.1890	927	859 329	30.4467	1002	1 004 004	31.6544	1077	1 159 929	32.8177
853	727 609	29.2062	928	861 184	30.4631	1003	1 006 009	31.6702	1078	1 162 084	32.8329
854	729 316	29.2233	929	863 041	30.4795	1004	1 008 016	31.6860	1079	1 164 241	32.8481
855	731 025	29.2404	930	864 900	30.4959	1005	1 010 025	31.7017	1080	1 166 400	32.8634
856	732 736	29.2575	931	866 761	30.5123	1006	1 012 036	31.7175	1081	1 168 561	32.8786
857	734 449	29.2746	932	868 624	30.5287	1007	1 014 049	31.7333	1082	1 170 724	32.8938
858	736 164	29.2916	933	870 489	30.5450	1008	1 016 064	31.7490	1083	1 172 889	32.9090
859	737 881	29.3087	934	872 356	30.5614	1009	1 018 081	31.7648	1084	1 175 056	32.9242
860	739 600	29.3258	935	874 225	30.5778	1010	1 020 100	31.7805	1085	1 177 225	32.9393
861	741 321	29.3428	936	876 096	30.5941	1011	1 022 121	31.7962	1086	1 179 396	32.9545
862	743 044	29.3598	937	877 969	30.6105	1012	1 024 144	31.8119	1087	1 181 569	32.9697
863	744 769	29.3769	938	879 844	30.6268	1013	1 026 169	31.8277	1088	1 183 744	32.9848
864	746 496	29.3939	939	881 721	30.6431	1014	1 028 196	31.8434	1089	1 185 921	33.0000
865	748 225	29.4109	940	883 600	30.6594	1015	1 030 225	31.8591	1090	1 188 100	33.0151
866	749 956	29.4279	941	885 481	30.6757	1016	1 032 256	31.8748	1091	1 190 281	33.0303
867	751 689	29.4449	942	887 364	30.6920	1017	1 034 289	31.8904	1092	1 192 464	33.0454
868	753 424	29.4618	943	889 249	30.7083	1018	1 036 324	31.9061	1093	1 194 649	33.0606
869	755 161	29.4788	944	891 136	30.7246	1019	1 038 361	31.9218	1094	1 196 836	33.0757
870	756 900	29.4958	945	893 025	30.7409	1020	1 040 400	31.9374	1095	1 199 025	33.0908
871	758 641	29.5127	946	894 916	30.7571	1021	1 042 441	31.9531	1096	1 201 216	33.1059
872	760 384	29.5296	947	896 809	30.7734	1022	1 044 484	31.9687	1097	1 203 409	33.1210
873	762 129	29.5466	948	898 704	30.7896	1023	1 046 529	31.9844	1098	1 205 604	33.1361
874	763 876	29.5635	949	900 601	30.8058	1024	1 048 576	32.0000	1099	1 207 801	33.1512
875	765 625	29.5804	950	902 500	30.8221	1025	1 050 625	32.0156	1100	1 210 000	33.1662

SQUARES AND SQUARE ROOTS (*Continued*) 1101–1400

No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root
1101	1 212 201	33.1813	1176	1 382 976	34.2929	1251	1 565 001	35.3695	1326	1 758 276	36.4143
1102	1 214 404	33.1964	1177	1 385 329	34.3074	1252	1 567 504	35.3836	1327	1 760 929	36.4280
1103	1 216 609	33.2114	1178	1 387 684	34.3220	1253	1 570 009	35.3977	1328	1 763 584	36.4417
1104	1 218 816	33.2265	1179	1 390 041	34.3366	1254	1 572 516	35.4119	1329	1 766 241	36.4555
1105	1 221 025	33.2415	1180	1 392 400	34.3511	1255	1 575 025	35.4260	1330	1 768 900	36.4692
1106	1 223 236	33.2566	1181	1 394 761	34.3657	1256	1 577 536	35.4401	1331	1 771 561	36.4829
1107	1 225 449	33.2716	1182	1 397 124	34.3802	1257	1 580 049	35.4542	1332	1 774 224	36.4966
1108	1 227 664	33.2866	1183	1 399 489	34.3948	1258	1 582 564	35.4683	1333	1 776 889	36.5103
1109	1 229 881	33.3017	1184	1 401 856	34.4093	1259	1 585 081	35.4824	1334	1 779 556	36.5240
1110	1 232 100	33.3167	1185	1 404 225	34.4238	1260	1 587 600	35.4965	1335	1 782 225	36.5377
1111	1 234 321	33.3317	1186	1 406 596	34.4384	1261	1 590 121	35.5106	1336	1 784 896	36.5513
1112	1 236 544	33.3467	1187	1 408 969	34.4529	1262	1 592 644	35.5246	1337	1 787 569	36.5650
1113	1 238 769	33.3617	1188	1 411 344	34.4674	1263	1 595 169	35.5387	1338	1 790 244	36.5787
1114	1 240 996	33.3766	1189	1 413 721	34.4819	1264	1 597 696	35.5528	1339	1 792 921	36.5923
1115	1 243 225	33.3916	1190	1 416 100	34.4964	1265	1 600 225	35.5668	1340	1 795 600	36.6060
1116	1 245 456	33.4066	1191	1 418 481	34.5109	1266	1 602 756	35.5809	1341	1 798 281	36.6197
1117	1 247 689	33.4215	1192	1 420 864	34.5254	1267	1 605 289	35.5949	1342	1 800 964	36.6333
1118	1 249 924	33.4365	1193	1 423 249	34.5398	1268	1 607 824	35.6090	1343	1 803 649	36.6470
1119	1 252 161	33.4515	1194	1 425 636	34.5543	1269	1 610 361	35.6230	1344	1 806 336	36.6606
1120	1 254 400	33.4664	1195	1 428 025	34.5688	1270	1 612 900	35.6371	1345	1 809 025	36.6742
1121	1 256 641	33.4813	1196	1 430 416	34.5832	1271	1 615 441	35.6511	1346	1 811 716	36.6879
1122	1 258 884	33.4963	1197	1 432 809	34.5977	1272	1 617 984	35.6651	1347	1 814 409	36.7015
1123	1 261 129	33.5112	1198	1 435 204	34.6121	1273	1 620 529	35.6791	1348	1 817 104	36.7151
1124	1 263 376	33.5261	1199	1 437 601	34.6266	1274	1 623 076	35.6931	1349	1 819 801	36.7287
1125	1 265 625	33.5410	1200	1 440 000	34.6410	1275	1 625 625	35.7071	1350	1 822 500	36.7423
1126	1 267 876	33.5559	1201	1 442 401	34.6554	1276	1 628 176	35.7211	1351	1 825 201	36.7560
1127	1 270 129	33.5708	1202	1 444 804	34.6699	1277	1 630 729	35.7351	1352	1 827 904	36.7696
1128	1 272 384	33.5857	1203	1 447 209	34.6843	1278	1 633 284	35.7491	1353	1 830 609	36.7831
1129	1 274 641	33.6006	1204	1 449 616	34.6987	1279	1 635 841	35.7631	1354	1 833 316	36.7967
1130	1 276 900	33.6155	1205	1 452 025	34.7131	1280	1 638 400	35.7771	1355	1 836 025	36.8103
1131	1 279 161	33.6303	1206	1 454 436	34.7275	1281	1 640 961	35.7911	1356	1 838 736	36.8239
1132	1 281 424	33.6452	1207	1 456 849	34.7419	1282	1 643 524	35.8050	1357	1 841 449	36.8375
1133	1 283 689	33.6601	1208	1 459 264	34.7563	1283	1 646 089	35.8190	1358	1 844 164	36.8511
1134	1 285 956	33.6749	1209	1 461 681	34.7707	1284	1 648 656	35.8329	1359	1 846 881	36.8646
1135	1 288 225	33.6898	1210	1 464 100	34.7851	1285	1 651 225	35.8469	1360	1 849 600	36.8782
1136	1 290 496	33.7046	1211	1 466 521	34.7994	1286	1 653 796	35.8608	1361	1 852 321	36.8917
1137	1 292 769	33.7194	1212	1 468 944	34.8138	1287	1 656 369	35.8748	1362	1 855 044	36.9053
1138	1 295 044	33.7343	1213	1 471 369	34.8281	1288	1 658 944	35.8887	1363	1 857 769	36.9188
1139	1 297 321	33.7491	1214	1 473 796	34.8425	1289	1 661 521	35.9026	1364	1 860 496	36.9324
1140	1 299 600	33.7639	1215	1 476 225	34.8569	1290	1 664 100	35.9166	1365	1 863 225	36.9459
1141	1 301 881	33.7787	1216	1 478 656	34.8712	1291	1 666 681	35.9305	1366	1 865 956	36.9594
1142	1 304 164	33.7935	1217	1 481 089	34.8855	1292	1 669 264	35.9444	1367	1 868 689	36.9730
1143	1 306 449	33.8083	1218	1 483 524	34.8999	1293	1 671 849	35.9583	1368	1 871 424	36.9865
1144	1 308 736	33.8231	1219	1 485 961	34.9142	1294	1 674 436	35.9722	1369	1 874 161	37.0000
1145	1 311 025	33.8378	1220	1 488 400	34.9285	1295	1 677 025	35.9861	1370	1 876 900	37.0135
1146	1 313 316	33.8526	1221	1 490 841	34.9428	1296	1 679 616	36.0000	1371	1 879 641	37.0270
1147	1 315 609	33.8674	1222	1 493 284	34.9571	1297	1 682 209	36.0139	1372	1 882 384	37.0405
1148	1 317 904	33.8821	1223	1 495 729	34.9714	1298	1 684 804	36.0278	1373	1 885 129	37.0540
1149	1 320 201	33.8969	1224	1 498 176	34.9857	1299	1 687 401	36.0416	1374	1 887 876	37.0675
1150	1 322 500	33.9116	1225	1 500 625	35.0000	1300	1 690 000	36.0555	1375	1 890 625	37.0810
1151	1 324 801	33.9264	1226	1 503 076	35.0143	1301	1 692 601	36.0694	1376	1 893 376	37.0945
1152	1 327 104	33.9411	1227	1 505 529	35.0286	1302	1 695 204	36.0832	1377	1 896 129	37.1080
1153	1 329 409	33.9559	1228	1 507 984	35.0428	1303	1 697 809	36.0971	1378	1 898 884	37.1214
1154	1 331 716	33.9706	1229	1 510 441	35.0571	1304	1 700 416	36.1109	1379	1 901 641	37.1349
1155	1 334 025	33.9853	1230	1 512 900	35.0714	1305	1 703 025	36.1248	1380	1 904 400	37.1484
1156	1 336 336	34.0000	1231	1 515 361	35.0856	1306	1 705 636	36.1386	1381	1 907 161	37.1618
1157	1 338 649	34.0147	1232	1 517 824	35.0999	1307	1 708 249	36.1525	1382	1 909 924	37.1753
1158	1 340 964	34.0294	1233	1 520 289	35.1141	1308	1 710 864	36.1663	1383	1 912 689	37.1887
1159	1 343 281	34.0441	1234	1 522 756	35.1283	1309	1 713 481	36.1801	1384	1 915 456	37.2022
1160	1 345 600	34.0588	1235	1 525 225	35.1426	1310	1 716 100	36.1939	1385	1 918 225	37.2156
1161	1 347 921	34.0735	1236	1 527 696	35.1568	1311	1 718 721	36.2077	1386	1 920 996	37.2290
1162	1 350 244	34.0881	1237	1 530 169	35.1710	1312	1 721 344	36.2215	1387	1 923 769	37.2424
1163	1 352 569	34.1028	1238	1 532 644	35.1852	1313	1 723 969	36.2353	1388	1 926 544	37.2559
1164	1 354 896	34.1174	1239	1 535 121	35.1994	1314	1 726 596	36.2491	1389	1 929 321	37.2693
1165	1 357 225	34.1321	1240	1 537 600	35.2136	1315	1 729 225	36.2629	1390	1 932 100	37.2827
1166	1 359 556	34.1467	1241	1 540 081	35.2278	1316	1 731 856	36.2767	1391	1 934 881	37.2961
1167	1 361 889	34.1614	1242	1 542 564	35.2420	1317	1 734 489	36.2905	1392	1 937 664	37.3095
1168	1 364 224	34.1760	1243	1 545 049	35.2562	1318	1 737 124	36.3043	1393	1 940 449	37.3229
1169	1 366 561	34.1906	1244	1 547 536	35.2704	1319	1 739 761	36.3180	1394	1 943 236	37.3363
1170	1 368 900	34.2053	1245	1 550 025	35.2846	1320	1 742 400	36.3318	1395	1 946 025	37.3497
1171	1 371 241	34.2199	1246	1 552 516	35.2987	1321	1 745 041	36.3456	1396	1 948 816	37.3631
1172	1 373 584	34.2345	1247	1 555 009	35.3129	1322	1 747 684	36.3593	1397	1 951 609	37.3765
1173	1 375 929	34.2491	1248	1 557 504	35.3270	1323	1 750 329	36.3731	1398	1 954 404	37.3898
1174	1 378 276	34.2637	1249	1 560 001	35.3412	1324	1 752 976	36.3868	1399	1 957 201	37.4032
1175	1 380 625	34.2783	1250	1 562 500	35.3553	1325	1 755 625	36.4005	1400	1 960 000	37.4166

1401–1700 SQUARES AND SQUARE ROOTS (*Continued*)

No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root
1401	1 962 801	37.4299	1476	2 178 576	38.4187	1551	2 405 601	39.3827	1626	2 643 876	40.3237
1402	1 965 604	37.4433	1477	2 181 529	38.4318	1552	2 408 704	39.3954	1627	2 647 129	40.3361
1403	1 968 409	37.4566	1478	2 184 484	38.4448	1553	2 411 809	39.4081	1628	2 650 384	40.3485
1404	1 971 216	37.4700	1479	2 187 441	38.4578	1554	2 414 916	39.4208	1629	2 653 641	40.3609
1405	1 974 025	37.4833	1480	2 190 400	38.4708	1555	2 418 025	39.4335	1630	2 656 900	40.3733
1406	1 976 836	37.4967	1481	2 193 361	38.4838	1556	2 421 136	39.4462	1631	2 660 161	40.3856
1407	1 979 649	37.5100	1482	2 196 324	38.4968	1557	2 424 249	39.4588	1632	2 663 424	40.3980
1408	1 982 464	37.5233	1483	2 199 289	38.5097	1558	2 427 364	39.4715	1633	2 666 689	40.4104
1409	1 985 281	37.5366	1484	2 202 256	38.5227	1559	2 430 481	39.4842	1634	2 669 956	40.4228
1410	1 988 100	37.5500	1485	2 205 225	38.5357	1560	2 433 600	39.4968	1635	2 673 225	40.4351
1411	1 990 921	37.5633	1486	2 208 196	38.5487	1561	2 436 721	39.5095	1636	2 676 496	40.4475
1412	1 993 744	37.5766	1487	2 211 169	38.5616	1562	2 439 844	39.5221	1637	2 679 769	40.4599
1413	1 996 569	37.5899	1488	2 214 144	38.5746	1563	2 442 969	39.5348	1638	2 683 044	40.4722
1414	1 999 396	37.6032	1489	2 217 121	38.5876	1564	2 446 096	39.5474	1639	2 686 321	40.4846
1415	2 002 225	37.6165	1490	2 220 100	38.6005	1565	2 449 225	39.5601	1640	2 689 600	40.4969
1416	2 005 056	37.6298	1491	2 223 081	38.6135	1566	2 452 356	39.5727	1641	2 692 881	40.5093
1417	2 007 889	37.6431	1492	2 226 064	38.6264	1567	2 455 489	39.5854	1642	2 696 164	40.5216
1418	2 010 724	37.6563	1493	2 229 049	38.6394	1568	2 458 624	39.5980	1643	2 699 449	40.5339
1419	2 013 561	37.6696	1494	2 232 036	38.6523	1569	2 461 761	39.6106	1644	2 702 736	40.5463
1420	2 016 400	37.6829	1495	2 235 025	38.6652	1570	2 464 900	39.6232	1645	2 706 025	40.5586
1421	2 019 241	37.6962	1496	2 238 016	38.6782	1571	2 468 041	39.6358	1646	2 709 316	40.5709
1422	2 022 084	37.7094	1497	2 241 009	38.6911	1572	2 471 184	39.6485	1647	2 712 609	40.5832
1423	2 024 929	37.7227	1498	2 244 004	38.7040	1573	2 474 329	39.6611	1648	2 715 904	40.5956
1424	2 027 776	37.7359	1499	2 247 001	38.7169	1574	2 477 476	39.6737	1649	2 719 201	40.6079
1425	2 030 625	37.7492	1500	2 250 000	38.7298	1575	2 480 625	39.6863	1650	2 722 500	40.6202
1426	2 033 476	37.7624	1501	2 253 001	38.7427	1576	2 483 776	39.6989	1651	2 725 801	40.6325
1427	2 036 329	37.7757	1502	2 256 004	38.7556	1577	2 486 929	39.7115	1652	2 729 104	40.6448
1428	2 039 184	37.7889	1503	2 259 009	38.7685	1578	2 490 084	39.7240	1653	2 732 409	40.6571
1429	2 042 041	37.8021	1504	2 262 016	38.7814	1579	2 493 241	39.7366	1654	2 735 716	40.6694
1430	2 044 900	37.8153	1505	2 265 025	38.7943	1580	2 496 400	39.7492	1655	2 739 025	40.6817
1431	2 047 761	37.8286	1506	2 268 036	38.8072	1581	2 499 561	39.7618	1656	2 742 336	40.6940
1432	2 050 624	37.8418	1507	2 271 049	38.8201	1582	2 502 724	39.7744	1657	2 745 649	40.7063
1433	2 053 489	37.8550	1508	2 274 064	38.8330	1583	2 505 889	39.7869	1658	2 748 964	40.7185
1434	2 056 356	37.8682	1509	2 277 081	38.8458	1584	2 509 056	39.7995	1659	2 752 281	40.7308
1435	2 059 225	37.8814	1510	2 280 100	38.8587	1585	2 512 225	39.8121	1660	2 755 600	40.7431
1436	2 062 096	37.8946	1511	2 283 121	38.8716	1586	2 515 396	39.8246	1661	2 758 921	40.7554
1437	2 064 969	37.9078	1512	2 286 144	38.8844	1587	2 518 569	39.8372	1662	2 762 244	40.7676
1438	2 067 844	37.9210	1513	2 289 169	38.8973	1588	2 521 744	39.8497	1663	2 765 569	40.7799
1439	2 070 721	37.9342	1514	2 292 196	38.9102	1589	2 524 921	39.8623	1664	2 768 896	40.7922
1440	2 073 600	37.9473	1515	2 295 225	38.9230	1590	2 528 100	39.8748	1665	2 772 225	40.8044
1441	2 076 481	37.9605	1516	2 298 256	38.9358	1591	2 531 281	39.8873	1666	2 775 556	40.8167
1442	2 079 364	37.9737	1517	2 301 289	38.9487	1592	2 534 464	39.8999	1667	2 778 889	40.8289
1443	2 082 249	37.9868	1518	2 304 324	38.9615	1593	2 537 649	39.9124	1668	2 782 224	40.8412
1444	2 085 136	38.0000	1519	2 307 361	38.9744	1594	2 540 836	39.9249	1669	2 785 561	40.8534
1445	2 088 025	38.0132	1520	2 310 400	38.9872	1595	2 544 025	39.9375	1670	2 788 900	40.8656
1446	2 090 916	38.0263	1521	2 313 441	39.0000	1596	2 547 216	39.9500	1671	2 792 241	40.8779
1447	2 093 809	38.0395	1522	2 316 484	39.0128	1597	2 550 409	39.9625	1672	2 795 584	40.8901
1448	2 096 704	38.0526	1523	2 319 529	39.0256	1598	2 553 604	39.9750	1673	2 798 929	40.9023
1449	2 099 601	38.0657	1524	2 322 576	39.0384	1599	2 556 801	39.9875	1674	2 802 276	40.9145
1450	2 102 500	38.0789	1525	2 325 625	39.0512	1600	2 560 000	40.0000	1675	2 805 625	40.9268
1451	2 105 401	38.0920	1526	2 328 676	39.0640	1601	2 563 201	40.0125	1676	2 808 976	40.9390
1452	2 108 304	38.1051	1527	2 331 729	39.0768	1602	2 566 404	40.0250	1677	2 812 329	40.9512
1453	2 111 209	38.1182	1528	2 334 784	39.0896	1603	2 569 609	40.0375	1678	2 815 684	40.9634
1454	2 114 116	38.1314	1529	2 337 841	39.1024	1604	2 572 816	40.0500	1679	2 819 041	40.9756
1455	2 117 025	38.1445	1530	2 340 900	39.1152	1605	2 576 025	40.0625	1680	2 822 400	40.9878
1456	2 119 936	38.1576	1531	2 343 961	39.1280	1606	2 579 236	40.0749	1681	2 825 761	41.0000
1457	2 122 849	38.1707	1532	2 347 024	39.1408	1607	2 582 449	40.0874	1682	2 829 124	41.0122
1458	2 125 764	38.1838	1533	2 350 089	39.1535	1608	2 585 664	40.0999	1683	2 832 489	41.0244
1459	2 128 681	38.1969	1534	2 353 156	39.1663	1609	2 588 881	40.1123	1684	2 835 856	41.0366
1460	2 131 600	38.2099	1535	2 356 225	39.1791	1610	2 592 100	40.1248	1685	2 839 225	41.0488
1461	2 134 521	38.2230	1536	2 359 296	39.1918	1611	2 595 321	40.1373	1686	2 842 596	41.0609
1462	2 137 444	38.2361	1537	2 362 369	39.2046	1612	2 598 544	40.1497	1687	2 845 969	41.0731
1463	2 140 369	38.2492	1538	2 365 444	39.2173	1613	2 601 769	40.1622	1688	2 849 344	41.0853
1464	2 143 296	38.2623	1539	2 368 521	39.2301	1614	2 604 996	40.1746	1689	2 852 721	41.0974
1465	2 146 225	38.2753	1540	2 371 600	39.2428	1615	2 608 225	40.1871	1690	2 856 100	41.1096
1466	2 149 156	38.2884	1541	2 374 681	39.2556	1616	2 611 456	40.1995	1691	2 859 481	41.1218
1467	2 152 089	38.3014	1542	2 377 764	39.2683	1617	2 614 689	40.2119	1692	2 862 864	41.1339
1468	2 155 024	38.3145	1543	2 380 849	39.2810	1618	2 617 924	40.2244	1693	2 866 249	41.1461
1469	2 157 961	38.3275	1544	2 383 936	39.2938	1619	2 621 161	40.2368	1694	2 869 636	41.1582
1470	2 160 900	38.3406	1545	2 387 025	39.3065	1620	2 624 400	40.2492	1695	2 873 025	41.1704
1471	2 163 841	38.3536	1546	2 390 116	39.3192	1621	2 627 641	40.2616	1696	2 876 416	41.1825
1472	2 166 784	38.3667	1547	2 393 209	39.3319	1622	2 630 884	40.2741	1697	2 879 809	41.1947
1473	2 169 729	38.3797	1548	2 396 304	39.3446	1623	2 634 129	40.2865	1698	2 883 204	41.2068
1474	2 172 676	38.3927	1549	2 399 401	39.3573	1624	2 637 376	40.2989	1699	2 886 601	41.2189
1475	2 175 625	38.4057	1550	2 402 500	39.3700	1625	2 640 625	40.3113	1700	2 890 000	41.2311

SQUARES AND SQUARE ROOTS (*Continued*) 1701–2000

No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root
1701	2 893 401	41.2432	1776	3 154 176	42.1426	1851	3 426 201	43.0232	1926	3 709 476	43.8862
1702	2 896 804	41.2553	1777	3 157 729	42.1545	1852	3 429 904	43.0349	1927	3 713 329	43.8976
1703	2 900 209	41.2674	1778	3 161 284	42.1663	1853	3 433 609	43.0465	1928	3 717 184	43.9090
1704	2 903 616	41.2795	1779	3 164 841	42.1782	1854	3 437 316	43.0581	1929	3 721 041	43.9204
1705	2 907 025	41.2916	1780	3 168 400	42.1900	1855	3 441 025	43.0697	1930	3 724 900	43.9318
1706	2 910 436	41.3038	1781	3 171 961	42.2019	1856	3 444 736	43.0813	1931	3 728 761	43.9431
1707	2 913 849	41.3159	1782	3 175 524	42.2137	1857	3 448 449	43.0929	1932	3 732 624	43.9545
1708	2 917 264	41.3280	1783	3 179 089	42.2256	1858	3 452 164	43.1045	1933	3 736 489	43.9659
1709	2 920 681	41.3401	1784	3 182 656	42.2374	1859	3 455 881	43.1161	1934	3 740 356	43.9773
1710	2 924 100	41.3521	1785	3 186 225	42.2493	1860	3 459 600	43.1277	1935	3 744 225	43.9886
1711	2 927 521	41.3642	1786	3 189 796	42.2611	1861	3 463 321	43.1393	1936	3 748 096	44.0000
1712	2 930 944	41.3763	1787	3 193 369	42.2729	1862	3 467 044	43.1509	1937	3 751 969	44.0114
1713	2 934 369	41.3884	1788	3 196 944	42.2847	1863	3 470 769	43.1625	1938	3 755 844	44.0227
1714	2 937 796	41.4005	1789	3 200 521	42.2966	1864	3 474 496	43.1741	1939	3 759 721	44.0341
1715	2 941 225	41.4126	1790	3 204 100	42.3084	1865	3 478 225	43.1856	1940	3 763 600	44.0454
1716	2 944 656	41.4246	1791	3 207 681	42.3202	1866	3 481 956	43.1972	1941	3 767 481	44.0568
1717	2 948 089	41.4367	1792	3 211 264	42.3320	1867	3 485 689	43.2088	1942	3 771 364	44.0681
1718	2 951 524	41.4488	1793	3 214 849	42.3438	1868	3 489 424	43.2204	1943	3 775 249	44.0795
1719	2 954 961	41.4608	1794	3 218 436	42.3556	1869	3 493 161	43.2319	1944	3 779 136	44.0908
1720	2 958 400	41.4729	1795	3 222 025	42.3674	1870	3 496 900	43.2435	1945	3 783 025	44 1022
1721	2 961 841	41.4849	1796	3 225 616	42.3792	1871	3 500 641	43.2551	1946	3 786 916	44.1135
1722	2 965 284	41.4970	1797	3 229 209	42.3910	1872	3 504 384	43.2666	1947	3 790 809	44.1248
1723	2 968 729	41.5090	1798	3 232 804	42.4028	1873	3 508 129	43.2782	1948	3 794 704	44.1362
1724	2 972 176	41.5211	1799	3 236 401	42.4146	1874	3 511 876	43.2897	1949	3 798 601	44.1475
1725	2 975 625	41.5331	1800	3 240 000	42.4264	1875	3 515 625	43.3013	1950	3 802 500	44.1588
1726	2 979 076	41.5452	1801	3 243 601	42.4382	1876	3 519 376	43.3128	1951	3 806 401	44.1701
1727	2 982 529	41.5572	1802	3 247 204	42.4500	1877	3 523 129	43.3244	1952	3 810 304	44.1814
1728	2 985 984	41.5692	1803	3 250 809	42.4617	1878	3 526 884	43.3359	1953	3 814 209	44.1928
1729	2 989 441	41.5812	1804	3 254 416	42.4735	1879	3 530 641	43.3474	1954	3 818 116	44.2041
1730	2 992 900	41.5933	1805	3 258 025	42.4853	1880	3 534 400	43.3590	1955	3 822 025	44.2154
1731	2 996 361	41.6053	1806	3 261 636	42.4971	1881	3 538 161	43.3705	1956	3 825 936	44.2267
1732	2 999 824	41.6173	1807	3 265 249	42.5088	1882	3 541 924	43.3820	1957	3 829 849	44.2380
1733	3 003 289	41.6293	1808	3 268 864	42.5206	1883	3 545 689	43.3935	1958	3 833 764	44.2493
1734	3 006 756	41.6413	1809	3 272 481	42.5323	1884	3 549 456	43.4051	1959	3 837 681	44.2606
1735	3 010 225	41.6533	1810	3 276 100	42.5441	1885	3 553 225	43.4166	1960	3 841 600	44.2719
1736	3 013 696	41.6653	1811	3 279 721	42.5558	1886	3 556 996	43.4281	1961	3 845 521	44.2832
1737	3 017 169	41.6773	1812	3 283 344	42.5676	1887	3 560 769	43.4396	1962	3 849 444	44.2945
1738	3 020 644	41.6893	1813	3 286 969	42.5793	1888	3 564 544	43.4511	1963	3 853 369	44.3058
1739	3 024 121	41.7013	1814	3 290 596	42.5911	1889	3 568 321	43.4626	1964	3 857 296	44.3170
1740	3 027 600	41.7133	1815	3 294 225	42.6028	1890	3 572 100	43.4741	1965	3 861 225	44.3283
1741	3 031 081	41.7253	1816	3 297 856	42.6146	1891	3 575 881	43.4856	1966	3 865 156	44.3396
1742	3 034 564	41.7373	1817	3 301 489	42.6263	1892	3 579 664	43.4971	1967	3 869 089	44.3509
1743	3 038 049	41.7493	1818	3 305 124	42.6380	1893	3 583 449	43.5086	1968	3 873 024	44.3621
1744	3 041 536	41.7612	1819	3 308 761	42.6497	1894	3 587 236	43.5201	1969	3 876 961	44.3734
1745	3 045 025	41.7732	1820	3 312 400	42.6615	1895	3 591 025	43.5316	1970	3 880 900	44.3847
1746	3 048 516	41.7852	1821	3 316 041	42.6732	1896	3 594 816	43.5431	1971	3 884 841	44.3959
1747	3 052 009	41.7971	1822	3 319 684	42.6849	1897	3 598 609	43.5546	1972	3 888 784	44.4072
1748	3 055 504	41.8091	1823	3 323 329	42.6966	1898	3 602 404	43.5660	1973	3 892 729	44.4185
1749	3 059 001	41.8210	1824	3 326 976	42.7083	1899	3 606 201	43.5775	1974	3 896 676	44.4297
1750	3 062 500	41.8330	1825	3 330 625	42.7200	1900	3 610 000	43.5890	1975	3 900 625	44.4410
1751	3 066 001	41.8450	1826	3 334 276	42.7317	1901	3 613 801	43.6005	1976	3 904 576	44.4522
1752	3 069 504	41.8569	1827	3 337 929	42.7434	1902	3 617 604	43.6119	1977	3 908 529	44.4635
1753	3 073 009	41.8688	1828	3 341 584	42.7551	1903	3 621 409	43.6234	1978	3 912 484	44.4747
1754	3 076 516	41.8808	1829	3 345 241	42.7668	1904	3 625 216	43.6348	1979	3 916 441	44.4860
1755	3 080 025	41.8927	1830	3 348 900	42.7785	1905	3 629 025	43.6463	1980	3 920 400	44.4972
1756	3 083 536	41.9047	1831	3 352 561	42.7902	1906	3 632 836	43.6578	1981	3 924 361	44.5084
1757	3 087 049	41.9166	1832	3 356 224	42.8019	1907	3 636 649	43.6692	1982	3 928 324	44.5197
1758	3 090 564	41.9285	1833	3 359 889	42.8135	1908	3 640 464	43.6807	1983	3 932 289	44.5309
1759	3 094 081	41.9404	1834	3 363 556	42.8252	1909	3 644 281	43.6921	1984	3 936 256	44.5421
1760	3 097 600	41.9524	1835	3 367 225	42.8369	1910	3 648 100	43.7035	1985	3 940 225	44.5533
1761	3 101 121	41.9643	1836	3 370 896	42.8486	1911	3 651 921	43.7150	1986	3 944 196	44.5646
1762	3 104 644	41.9762	1837	3 374 569	42.8602	1912	3 655 744	43.7264	1987	3 948 169	44.5758
1763	3 108 169	41.9881	1838	3 378 244	42.8719	1913	3 659 569	43.7379	1988	3 952 144	44.5870
1764	3 111 696	42.0000	1839	3 381 921	42.8836	1914	3 663 396	43.7493	1989	3 956 121	44.5982
1765	3 115 225	42.0119	1840	3 385 600	42.8952	1915	3 667 225	43.7607	1990	3 960 100	44.609[illegible]
1766	3 118 756	42.0238	1841	3 389 281	42.9069	1916	3 671 056	43.7721	1991	3 964 081	44.620
1767	3 122 289	42.0357	1842	3 392 964	42.9185	1917	3 674 889	43.7836	1992	3 968 064	44.631[illegible]
1768	3 125 824	42.0476	1843	3 396 649	42.9302	1918	3 678 724	43.7950	1993	3 972 049	44.643[illegible]
1769	3 129 361	42.0595	1844	3 400 336	42.9418	1919	3 682 561	43.8064	1994	3 976 036	44.6542
1770	3 132 900	42.0714	1845	3 404 025	42.9535	1920	3 686 400	43.8178	1995	3 980 025	44.6654
1771	3 136 441	42.0833	1846	3 407 716	42.9651	1921	3 690 241	43.8292	1996	3 984 016	44.6766
1772	3 139 984	42.0951	1847	3 411 409	42.9767	1922	3 694 084	43.8406	1997	3 988 009	44.6878
1773	3 143 529	42.1070	1848	3 415 104	42.9884	1923	3 697 929	43.8520	1998	3 992 004	44.6990
1774	3 147 076	42.1189	1849	3 418 801	43.0000	1924	3 701 776	43.8634	1999	3 996 001	44.7102
1775	3 150 625	42.1307	1850	3 422 500	43.0116	1925	3 705 625	43.8748	2000	4 000 000	44.7214

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Jour. Am. Stat. Assn.
Jour. Royal Stat. Soc.—B
**Technometrics*

Pamphlets:

*ASA Standard Z1.1-1958, "Guide for Quality Control" (ASQC Std B1-1958)

*ASA Standard Z1.2-1958, "Control Chart Method of Analyzing Data" (ASQC Std B2-1958)

*ASA Standard Z1.3-1958, "Control Chart Method of Controlling Quality During Production" (ASQC Std B3-1958)

*ASQC General Publications

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* With special reference to quality control.

COMPARISON OF SYMBOLS

TABLE OF SOME SYMBOLS USED IN THE MANUAL AND THOSE USED IN STATISTICAL TEXTS

NOTE.—The Manual uses the prime notation for universe parameters (or standard values), while statistical texts are inclined to use Greek letters.

Term	Symbol Used in the Manual	Symbol Commonly Used in Statistical Texts
An observed value	X	X (or x)
Universe mean or average	$\bar{X}'$	μ
Sample size, or number of observations	n	n (or N)
Sample mean or average	$\bar{X}\left(=\frac{\Sigma X_i}{n}\right)$	$\bar{X}$ (or $\bar{x}$)
Universe standard deviation	σ'	σ
Sample standard deviation	$\sigma\left(=\sqrt{\frac{\Sigma(X_i-\bar{X})^2}{n}}\right)$	$s\left(=\sqrt{\frac{\Sigma(X_i-\bar{X})^2}{n-1}}\right)^a$
Universe variance	σ'^2	σ^2
Sample variance	$\sigma^2\left(=\frac{\Sigma(X_i-\bar{X})^2}{n}\right)$	$s^2\left(=\frac{\Sigma(X_i-\bar{X})^2}{n-1}\right)^a$

[a] Some authorities feel the term "sample standard deviation" for s and the term "sample variance" for s^2 to be misapplied. In any case s^2 is the unbiased estimate of the universe variance.

Recommended Practice for

CHOICE OF SAMPLE SIZE TO ESTIMATE THE AVERAGE QUALITY OF A LOT OR PROCESS[1]

ASTM Designation: E 122 – 58

ADOPTED, 1958.[2]

This Recommended Practice of the American Society for Testing Materials is issued under the fixed designation E 122; the final number indicates the year of original adoption or, in the case of revision, the year of last revision.

NOTE.—Note 3 of Section 4 (*a*) was formerly Example 1. All subsequent notes and examples were accordingly renumbered editorially in July, 1958.

Scope

1. This recommended practice presents simple methods for calculating how many units to include in a sample in order to estimate, with a prescribed precision, the average of some characteristic for all the units of a lot of material, or the average produced by a process.

Empirical Knowledge Needed

2. (*a*) Some empirical knowledge of the problem is necessary as follows:

(*1*) The standard deviation or, if that is not possible,

(*2*) The range or spread of the characteristic, from its lowest to its highest value and, if possible, some knowledge of the shape of the distribution of the characteristic; for instance, whether most of the values lie at one end of the range, or are mostly in the middle, or run rather uniformly from one end to the other.

(*b*) If the aim is to estimate the fraction defective, then each unit has a value of 0 or 1 (not defective or defective), and the standard deviation, as well as the shape, of the distribution depends only on p', the fraction defective of the lot or process.

(*c*) Sketchy knowledge is sufficient to start on, although more knowledge permits greater economy in the sample. Rarely will there be difficulty in acquiring enough information to compute the required size of sample with sufficient assurance beforehand to meet the desired precision within acceptable limits. A sample that is bigger than the equations indicate is used in actual practice when the empirical knowledge is only sketchy to start with, and if the desired precision is critical. The extra insurance is the price of incomplete knowledge.

(*d*) In any case, even when starting with sketchy knowledge, the precision of the estimate made from a random sample

[1] Under the standardization procedure of the Society, this recommended practice is under the jurisdiction of the ASTM Committee E-11 on Quality Control of Materials.

[2] Prior to adoption, this recommended practice was published as tentative from 1956 to 1958.

may itself be estimated from the sample. This estimation of the precision reached by the first sample makes it possible to fix more economically the sample size for the next sample of a similar material. In other words, information concerning the process, and the material produced thereby, accumulates and should be used.

Precision Desired

3. The approximate precision desired for the estimate must be prescribed. That is, it must be decided what maximum difference, E, can be tolerated between the estimate to be made from the sample and the result that would be obtained by testing every unit in the universe.

Equations for Calculating Sample Size

4. (*a*) The equation for the size n of the sample is as follows:

$$n = \left(\frac{3\sigma'}{E}\right)^2 \quad \text{.....(1)}$$

where:

σ' = the advance estimate of the standard deviation of the lot or process. (Note 1).

NOTE 1.—Some simple methods are given later to show how to reduce the empirical knowledge to the numerical value σ'.

E = the maximum allowable difference between the estimate to be made from the sample and the result of testing (by the same methods) all the units in the universe.

3 = a factor corresponding to a probability of about 3 parts in 1000 (Note 2) that the difference between the sample estimate and the result of testing (by the same methods) all the units in the universe is greater than E. The choice of the factor 3 is recommended for general use. With the factor 3, and with a universe standard deviation equal to the advance estimate, it is "practically certain" that the sampling error will not exceed E. There are occasions, however, where a lesser degree of certainty is desired, which a smaller factor provides (Note 3).

NOTE 2.—In the sampling of a lot of material that has a highly skewed distribution in the characteristic measured, the factor 3 will give a different probability, possibly as great as 6 parts in 1000. If there is anxiety about the effect of skewness, there are two things which can be done:

(*1*) Probe the material with a view to discovering, for example, extra-high values, or possibly spotty runs of abnormal character, in order to approximate roughly the amount of the skewness, for use with statistical theory and adjustment of the sample size if necessary.

(*2*) Search the lot for abnormal material and segregate it for separate treatment.

NOTE 3.—For example, the factor 2 gives a probability of about 45 parts in 1000 that the sampling error will exceed E. Although the distributions met in practice may not be normal, the following table (based on the normal distribution) indicates approximate probabilities:

Factor	Probability
3	3 in 1000
2	45 in 1000
2.58	1 in 100
1.96	5 in 100 (1 in 20)
1.64	10 in 100 (1 in 10)

(*b*) It is sometimes convenient to use Eq 1 in another form: namely,

$$n = \left(\frac{3v'}{e}\right)^2 \quad \text{.....(2)}$$

where:

v' (coefficient of variation in per cent) = $100\,\sigma'/\overline{X}'$, the advance estimate of the coefficient of variation of the material, expressed in per cent

e = $100\,E/\overline{X}'$, the allowable sampling error expressed as a per cent of $\overline{X}'$, and

$\overline{X}'$ = the expected value of the characteristic being measured.

There are some materials for which σ' varies approximately with $\overline{X}'$, in which

case v' remains approximately constant from large to small values of $\overline{X}'$. If the relative error, e, is to be the same for all values of $\overline{X}'$, then everything on the right-hand side of Eq 2 is a constant; hence n is also a constant, which means that the same sample size, n, would be required for all sizes of $\overline{X}'$.

(*c*) If the problem is to estimate the fraction defective, then $(\sigma')^2$ is replaced by $p'(1 - p')$, so that Eq 1 becomes:

$$n = \left(\frac{3}{E}\right)^2 p'(1 - p') \quad \text{.....(3)}$$

where:

p' = the advance estimate of the fraction defective.

If p' is small, so that $p'n$ is less than 4, then 3.25 should be used in place of 3 in Eq 3 to compensate for the skewness of the p distribution for small values of p'.

(*d*) When the average of a particular lot of limited size is wanted, and an estimate of the average for the process is not part of the problem, the required sample size is less than Eqs 1, 2, and 3 indicate. The sample size for estimating the average of the finite lot will be:

$$n_L = \left(\frac{N}{N + n}\right) n \quad \text{.....(4)}$$

where:

n = the value computed from Eqs 1, 2, or 3, and

N = the lot size.

This reduction in size is usually of little importance unless n is 10 per cent or more of N.

Reduction of Empirical Knowledge to a Numerical Value of σ' (Data for Previous Samples Available)

5. (*a*) This section illustrates the use of the equations in Section 4 when there are data for previous samples.

(*b*) *For Equation 1.*—Compute the standard deviation σ (corrected for sample size)[3] for several samples, and use the average of them, if they are not too dissimilar, for an advance estimate of σ'.

NOTE 4.—A simple way to compute the overall σ for a lot is to arrange the observed values in a random order, and then average the ranges of successive groups of 4, 5, 8, or 10 observed values. (Theory shows that the optimum subgroup size is 8.) If $\overline{R}$ is the average of these ranges, then $\overline{R}/d_2$ is an estimate of σ.

The accompanying table shows some selected values[3] of d_2.

Group Size	d_2
2	1.13
4	2.06
5	2.33
8	2.85
10	3.08

Example 1.—*Use of* σ:

Problem.—To compute the sample size needed to estimate the average transverse strength of a lot of bricks when the desired value of E is 50 psi.

Solution.—From the data of three previous lots, the values of standard deviation were found to be 215, 192, and 202 psi, based on samples of 100 bricks. The average of these three standard deviations is 204 psi, whence Eq 1 gives:

$$n = \left(\frac{3 \times 204}{50}\right)^2 = (12.2)^2 = 148.8$$

$$= 149 \text{ bricks}$$

for the required size of sample to give a maximum sampling error of 50 psi.

(*c*) *For Equation 2.*—If σ' varies approximately with $\overline{X}'$ for the characteristic of the material to be measured, compute both the average, $\overline{X}$, and the standard deviation, σ (corrected for sample size),[3] for several samples (unless they are already available). An average of the several values of $v = \sigma/\overline{X}$, if they are not too dissimilar, may be used as an advance estimate of v'.

Example 2.—*Use of* v:

Problem.—To compute the sample size needed to estimate the average abrasion resistance of a material when the desired value of e is 10 per cent.

[3] See the ASTM Manual on Quality Control of Materials, *STP 15-C*, p. 63, for values of c_2 for correcting σ when n is less than 25, as well as for values of d_2.

Solution.—There are no data from previous samples of this same material, but data for six samples of similar materials show a wide range of resistance. However, the values of standard deviation are approximately proportional to the observed averages, as shown in the following table:

Lot No.	Sample Size	Average Cycles	Observed Range, R	$\sigma = \frac{R}{(3.08)}$ *	Coefficient of Variation, v, in per cent
1.....	10	90	40	13.0	14
2.....	10	190	100	32.5	17
3.....	10	350	140	45.5	13
4.....	10	450	220	71.4	16
5.....	10	1000	360	116.9	12
6.....	10	3550	2090	678.6	19
Avg.....					15.2

* Values of standard deviation (corrected for sample size) may be used instead of the estimates made from the range, if they are preferred or already available.

The use of the average of the observed values of v as an advance estimate of v' in Eq 2 gives:

$$n = \left(\frac{3 \times 15.2}{10}\right)^2 = (4.6)^2 = 21.2$$

$$= 22 \text{ specimens}$$

for the required size of sample to give a maximum sampling error of 10 per cent of the expected value.

If a maximum allowable error of 5 per cent were needed, the required sample size would be 85 specimens. The data supplied by the prescribed sample will be useful for the next investigation of similar material.

(*d*) *For Equation 3.*—Compute the value of fraction defective, p, for each sample. If the values are not too dissimilar, use the average of them for an advance estimate of p'. (If the sample sizes vary, use a weighted average

$$\bar{p} = \frac{\text{total number of defectives in all samples}}{\text{total number of units in all samples}}$$

instead of a simple average of the p values.) If the values are quite dissimilar, decide whether to use some of them to obtain an advance estimate of p'.

Example 3.—*Use of p:*

Problem.—To compute the sample size needed to estimate the fraction defective in a lot of alloy steel track bolts and nuts when the desired value of E is 0.04.

Solution.—The following data from four previous lots were used for an advance estimate of p':

Lot No.	Sample Size	No. of Defectives	Fraction Defective
1.....	75	3	0.040
2.....	100	10	0.100
3.....	90	4	0.044
4.....	125	4	0.032
Total.....	390	21	

$$\bar{p} = \frac{21}{390} = 0.054$$

$$n = \left(\frac{3}{0.04}\right)^2 (0.054)(0.946)$$

$$= \frac{9 \times 0.0511}{0.0016} = 287.4 = 288$$

If the desired value of E were 0.01, the required sample size would be 4600. It would be smaller if Eq 4 applies.

Reduction of Empirical Knowledge to a Numerical Value of σ' (No Data from Previous Samples of the Same or Like Material Available)

6. (*a*) This section illustrates the use of the equations in Section 4 when there are no actual observed values for the computation of σ.

(*b*) *For Equation 1.*—From past experience, estimate what the smallest and largest values of the characteristic are likely to be. If this is not known, obtain this information from some other source. Try to picture how the other observed values are probably distributed. A few simple observations and questions concerning the past behavior of the process, the usual procedure of blending, mixing, stacking, storing, etc., and concerning the aging of material and the usual prac-

tice of withdrawing the material (last in, first out; or last in, last out) will usually elicit sufficient information to distinguish between one triangular distribution and another in Fig. 1. In case of doubt, or in case the desired precision E is a critical matter, the rectangular distribution may be used. The price of the extra protection afforded by the rectangular distribution is a larger sample size, owing to the larger standard deviation thereof. At the worst, if the isosceles triangle is used when the other triangle or the rectangle is a better description, then the standard error of the result is larger by no more than 40 per cent, as shown by comparing the formulas for the standard deviations given in Fig. 1. The sizes of subsequent samples may then be adjusted upward, if necessary.

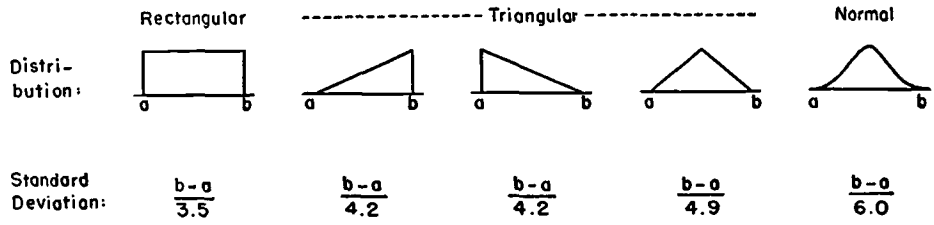


FIG. 1.—Some Types of Distributions and Their Standard Deviations.

NOTE 5.—The standard deviation of the normal distribution in Fig. 1 is a safe assumption for materials with a good history of control, in which case an advance estimate of σ' would usually be available.

The standard deviation estimated from one of the formulas of Fig. 1 may be used as an advance estimate of σ' in Eq 1. This method of advance estimation is in constant use and is often preferable to doubtful observed values.

Example 4.—*Use of σ from Fig. 1:*

Problem (*Same as Example 1*).—To compute the sample size needed to estimate the average transverse strength of a lot of bricks when the desired value of E is 50 psi.

Solution.—From past experience the range of values of transverse strength for a lot of bricks has been about 1200 psi. The values were heaped up in the middle of this range, but not necessarily normally distributed.

The isosceles triangle in Fig. 1 appears to be most appropriate; the advance estimate of σ' is 1200/4.9 = 245 psi. Then:

$$n = \left(\frac{3 \times 245}{50}\right)^2 = (14.7)^2 = 216.1$$

$$= 217 \text{ bricks}$$

The difference between 217 and 149 bricks (found in Example 1) is the price of sketchy knowledge.

(*c*) *For Equation 2.*—While the estimation of the coefficient of variation of a universe by use of Fig. 1 is possible, it is not recommended. In general, the knowledge that the use of v', instead of σ', is preferable would be obtained from the analysis of actual data, in which case the methods of Section 5 apply.

(*d*) *For Equation 3.*—From past experience, estimate approximately the range within which the fraction defective is likely to lie. Turn to Fig. 2 and read off the value of $\sigma^2 = p(1 - p)$ for the middle of the possible range of p, and use it in Eq 3. In case the desired precision is a critical matter, use the largest value of σ^2 within the possible range of p.

Consideration of Cost

7. (*a*) After the required size of sample to meet a prescribed precision is computed from Eqs 1, 2, or 3, the next step is to compute the cost of testing this

size of sample. If the cost is too great, it may be possible to relax the required precision (or the equivalent, which is to accept an increase in the probability (Section 4) that the sampling error may exceed the maximum allowable error, E) and to reduce the size of the sample to meet the allowable cost.

(*b*) As an alternative to Eq 1, which gives n in terms of a prescribed precision, this equation may be solved for E in terms of n, thus discovering what precision is possible for a given allowable cost. The same may be done for Eqs 2 and 3.

(*c*) It is necessary to specify either the desired allowable error, E, or the allowable cost; otherwise there is no proper size of sample.

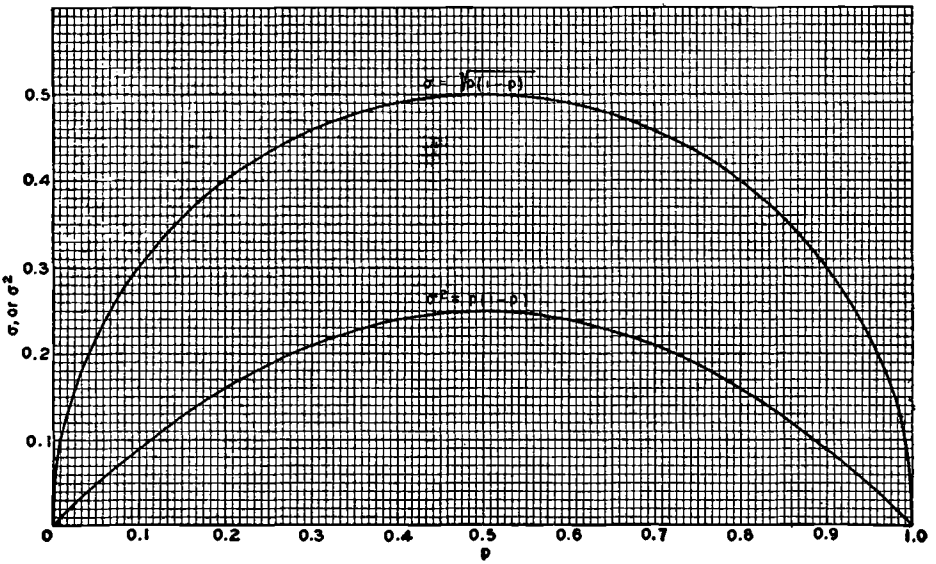


FIG. 2.—Values of σ, or σ^2, Corresponding to Values of p.

Selection of the Sample

8. (*a*) In order to make any estimate for a lot or for a process, on the basis of a sample, it is necessary to select the units in the sample "at random." Randomness is not just accident or lack of direction; it is the product of a definite and willful effort to produce disorder. The only universally acceptable definition of a random selection is by the use of random numbers, which are in effect the guarantee of thorough stirring of the sampling units in a lot.

(*b*) In the use of random numbers, the material must first be broken up in some manner into "sampling units." Moreover, each sampling unit is identifiable by a serial number, actual or by rule. For packaged articles, a rule is easy; the package contains a certain number of articles in definite layers, arranged in a particular way, and it is easy to devise some system for numbering the articles. In the case of bulk material like ore, or coal, or a barrel of bolts or nuts, the problem of defining usable sampling units must take place at an earlier stage of manufacture.

(*c*) It is not the purpose here to discuss the handling of materials, nor to find ways by which one can with surety discover the way to a satisfactory type of sampling unit. Instead, the aim is to assume that a suitable sampling unit has

been defined, and then to answer the question of how many to draw.

Estimation of the Precision from the Results of the Sample

9. (*a*) Equation 1 is a prediction and is used to compute the required size of sample. However, after the sample has been tested, the actual value of the maximum sampling error, E, may be estimated. One procedure for computing the value of the standard deviation of the sample is that given in Section 5. Then the estimate of the maximum difference between the sample estimate and the result of testing (by the same methods) all the units in the universe is:

$$E_{est} = \frac{3\bar{R}}{d_2\sqrt{n}} \quad \text{.....(5)}$$

where:

E_{est} = the sample estimate of E, and
n = the total sample size.

(*b*) When the sample is not apportioned by strata as described in Section 10, an equivalent estimate of the maximum sampling error is:

$$E_{est} = \frac{3\sigma}{\sqrt{n-1}} \quad \text{.....(6)}$$

where:

$\sigma = \sqrt{\frac{\Sigma X^2}{n} - \bar{X}^2}$, as given in the ASTM Manual on Quality Control of Materials, Part 1.[4]

NOTE 6.—If n is large, either estimate will be reliable. If n is small, either estimate will be subject to a wide sampling error, and may not be as reliable as the advance estimate made from prior information. The following table indicates the sampling error in σ for samples of size n drawn from a normal population whose standard deviation is σ'. The following values[5] for the ratio σ/σ' will be exceeded by chance alone about 5 times in 100:

Sample Size	σ/σ'
5	1.378
10	1.301
15	1.257
20	1.228
25	1.207
30	1.191
50	1.152
100	1.110
200	1.079

(*c*) In estimating a fraction defective, one should remember that the estimate is subject to sampling error, the maximum of which will be:

$$E_{est} = 3\,\frac{\sqrt{p(1-p)}}{\sqrt{n}} \quad \text{..... (7)}$$

$\sqrt{p(1-p)}$ may be read from Fig. 2.

If p is small, so that pn is less than 4, then 3.25 should be used in place of 3 in Eq 7, to compensate for the skewness of the p distribution for small values of p.

Sampling by Sub-lots or by Strata

10. It is advisable, and sometimes easier, to apportion the sample by strata (sub-lots, layers, sheets, or other natural divisions), as theory shows that such a plan will occasionally show gains in precision. It is important, in the use of Eq 5 for this kind of sampling, to average the ranges of the strata, as otherwise Eq 5 will overestimate the sampling error.

[4] See p. 16 of the ASTM Manual, *STP 15-C.*

[5] F. E. Croxton and D. J. Cowden, *Industrial Quality Control*, Vol. 3, July, 1946, pp. 18–21.

American Society for Testing and Materials

1916 Race Street Philadelphia 3, Pa.

Application for Membership

The undersigned hereby applies for membership (which includes subscription(s) to *Materials Research & Standards*) in the American Society for Testing and Materials in the class of:

☐ Sustaining ☐ Industrial ☐ Institutional

☐ Personal ☐ Associate

and if elected to membership agrees to be governed by the Charter and By-laws of the Society and to further its objectives as laid down therein.

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(Firm, organization or person)

OFFICIAL REPRESENTATIVE.....

(If sustaining, industrial or institutional membership, indicate the name and title of individual who will exercise membership privileges.)

TITLE AND DEPT.....

NAME OF ORGANIZATION.....

(If personal or associate membership, indicate the name of organization with which applicant is affiliated.)

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NATURE OF BUSINESS.....

ADDRESS FOR MAIL.....

(if other than above)

CITY.....ZONE.....STATE.....

DATE OF BIRTH.....

GRADUATE OF, OR ATTENDED.....

(Name of college or university)

YEAR.....Degree, or Course.....

SIGNATURE.....

Proposed by

(Two Members).....

.....

A list of ASTM members in your locality will be furnished on request.

AMERICAN SOCIETY FOR TESTING AND MATERIALS

EXTRACT FROM CHARTER

1. The name of the proposed corporation is the "American Society for Testing and Materials."
2. The corporation is formed for the promotion of knowledge of the materials of engineering, and the standardization of specifications and the methods of testing.

EXTRACT FROM BY-LAWS

ARTICLE I. *Members and Their Election*

SECTION 1. The corporate membership of the Society shall consist of Personal Members, Institutional Members, Industrial Members, Sustaining Members, Associate Members, and Honorary Members elected from a corporate grade of membership. In addition there shall be Student Members, and Honorary Members elected from nonmembers of the Society. The rights of membership of Institutional, Industrial, and Sustaining Members shall be exercised by the individual who is designated as the official representative of that membership.

SECTION 2. A Personal Member shall be a person meeting the qualifications established by the Board of Directors for this classification.

SECTION 3. An Institutional Member shall be a public library; educational institution; a non-profit professional, scientific or technical society; government department or agency at the federal, state, city, county or township level; or separate divisions thereof meeting the qualifications established by the Board of Directors for this classification.

SECTION 4. An Industrial Member shall be a plant, firm, corporation, partnership, or other business enterprise, or separate divisions thereof; trade association, or research institute meeting the qualifications established by the Board of Directors for this classification.

SECTION 5. A Sustaining Member shall be a person, plant, firm, corporation, society, department of government or other organization, or separate divisions thereof, electing to give greater support to the Society's activities through the payment of larger dues.

SECTION 6. An Associate Member shall be a person less than thirty years of age. He shall have the same rights and privileges as a Personal Member, except that he shall not be eligible for office. An Associate Member shall not remain in this category beyond the end of the calendar year in which his thirtieth birthday occurs.

ARTICLE V. *Meetings*

SECTION 1. The Society shall meet annually, for the transaction of its business, at a time and place fixed by the Board of Directors. Twenty-five corporate members shall constitute a quorum.

SECTION 2. Special business meetings of the Society may be called at any time and place at the discretion of the Board of Directors, or shall be called by the President, upon the written request of at least one per cent of the Corporate Membership.

ARTICLE VIII. *Dues*

SECTION 1. The membership year shall commence on the first day of January. The annual dues*, payable in advance, shall be as follows: For Personal Members, $18; for Institutional Members, $25; for Industrial Members, $75; for Sustaining Members, $200; for Associate Members, $10; for Student Members, $3. Honorary Members shall not be subject to dues.

SECTION 2. The entrance fees, payable on admission to the Society, shall be $10 for Personal Members, Institutional Members, Industrial Members and Sustaining Members, and $5 for Associate Members. Student Members shall pay no entrance fee. There shall be no fee for transfer from one class of membership to another.

SECTION 6. Any person elected after six months of any membership year shall have expired, may pay only one-half of the amount of dues for that year.

*NOTE—Of the annual dues $5.00 is for subscription to MATERIALS RESEARCH & STANDARDS.