E 1921 Appendix

(Nonmandatory Information copied directly from appendixes in E1921)

X1. WEIBULL FITTING OF DATA

X1.1 *Description of the Weibull Model*:

X1.1.1 The three-parameter Weibull model is used to fit the relationship between K_{Jc} and the cumulative probability for failure, p_f. The term p_f is the probability for failure at or before K_{Jc} for an arbitrarily chosen specimen from the population of specimens. This can be calculated from the following:

$$P_f = 1 - exp\ \{-[(K_{Jc} - K_{min})/(K_o - K_{min})]^b\} \qquad \text{(X1.1)}$$

X1.1.2 Ferritic steels of yield strengths ranging from 275 to 825 MPa (40 to 120 ksi) will have fracture toughness distributions of nearly the same shape when K_{min} is set at 20 MPa$\sqrt{m}$ (18.2 ksi$\sqrt{in.}$). This shape is defined by the Weibull exponent, b, which is constant at 4. Scale parameter, K_o, is a data-fitting parameter. The procedure is described in X1.2.

X1.2 *Determination of Scale Parameter, K_o, and Median K_{Jc}*—The following example illustrates the use of 10.2.1. The data came from tests that used 4T compact specimens of A533 grade B steel tested at −75°C. All data are valid and the chosen equivalent specimen size for analysis will be 1T.

Rank (i)	$K_{Jc(4T)}$ (MPa$\sqrt{m}$)	$K_{Jc(1T)}$ Equivalent (MPa$\sqrt{m}$)
1	59.1	75.3
2	68.3	88.3
3	77.9	101.9
4	97.9	130.2
5	100.9	134.4
6	112.4	150.7

$$K_{o(1T)} = \left[\sum_{i=1}^{N} \frac{\left(K_{Jc(i)} - 20\right)^4}{N} \right]^{1/4} + 20 \qquad \text{(X1.2)}$$

$$N = 6$$

$$K_{o(1T)} = 123.4 \text{ MPa}\sqrt{m}$$

X1.2.1 Median K_{Jc} is obtained as follows:

$$K_{Jc(med)} = 20 + (K_{o(1T)} - 20)(0.9124)\ \mathrm{MPa}\sqrt{\mathrm{m}} \tag{X1.3}$$

$$= 114.4\ \mathrm{MPa}\sqrt{\mathrm{m}}$$

X1.2.2

$$T_o = T - \left(\frac{1}{0.019}\right) ln\left[\frac{K_{Jc\,(med)} - 30}{70}\right] \tag{X1.4}$$

$$= -85°C$$

X1.3 *Data Censoring Using the Maximum Likelihood Method*:

X1.3.1 *Censoring When $K_{Jc(limit)}$ is Violated*—The following example uses 10.2.2 where all tests have been made at one test temperature. The example data set is artificially generated for a material that has a T_o reference temperature of 0°C. Two specimen sizes are 1/2T and 1T with six specimens of each size. Invalid K_{Jc} values and their dummy replacement $K_{Jc(limit)}$ values will be within parentheses.

X1.3.2 The data distribution is developed with the following assumptions:

Material yield strength = 482 MPa or 70 ksi
T_o temperature = 0°C
Test temperature = 38°C
1/2T and 1T specimens; all a/W = 0.5

X1.3.3 $K_{Jc(limit)}$ values in MPa$\sqrt{\mathrm{m}}$ from Eq. 1.

	0.5T	1T
Specimen size	206	291
1T equivalent	176	291

X1.3.4 *Simulated Data Set*:

Raw Data (K_{Jc}, MPa$\sqrt{\mathrm{m}}$)		Size Adjusted ($K_{Jc(1T)}$, MPa$\sqrt{\mathrm{m}}$)	
1/2T	1T	1/2T ^{A}	1T
138.8	119.9	119.9	119.9
171.8	147.6	147.6	147.6
195.2	167.3	167.3	167.3
(216.2)	185.0	(176)	185.0
(238.5)	203.7	(176)	203.7
(268.3)	228.8	(176)	228.8

^{A} $K_{Jc(1T)} = (K_{Jc(0.5T)} - 20)\ (1/2\ /\ 1)^{1/4} + 20$ MPa$\sqrt{\mathrm{m}}$

$$K_{o(1T)} = \left[\sum_{i-1}^{N} \frac{(K_{Jc(i)} - 20)^4}{r}\right]^{1/4} + 20 \tag{X1.5}$$

where:

N = 12,
r = 9,
$K_{o(1T)}$ = 188 MPa$\sqrt{\text{m}}$,
$K_{Jc(med)}$ = 174 MPa$\sqrt{\text{m}}$, and
T_o = 0°C.

X1.3.5 *Censoring When* $\Delta a_p \leq 0.05(W - a_p)$, *not to Exceed 1 mm Limit is Violated*—The following example uses 10.2.2 where all tests have been made at a single test temperature of 38°C. Assume that the test material has properties as defined in X1.3.2 and toughness data as defined in X1.3.4. However, for this example assume that the steel has a low upper shelf. The crack growth limit (see 8.9.2) is 0.64 mm and 1 mm for 0.5T and 1T specimen respectively. The K_J value after 0.64 mm of slow-stable growth is only 197 MPa$\sqrt{\text{m}}$ and after 1 mm of slow-stable growth is only 202 MPa$\sqrt{\text{m}}$. Therefore, the crack growth limit controls all censoring. The K_j–R curve is specimen size independent so that both 0.5T and 1T specimens will have censored data. In this case the dummy replacement value as per 10.2.2 is the highest ranked valid K_{Jc} value.

Raw Data				1T Size Adjusted Data	
0.5T		1T		0.5T ^{A}	1T
Δa_p, mm	K_{Jc}, Mpa$\sqrt{\text{m}}$	Δa_p, mm	K_{Jc}, Mpa$\sqrt{\text{m}}$	K_{Jc}, Mpa$\sqrt{\text{m}}$	
0.00	138.8	0.00	119.9	119.9	119.9
0.25	171.8	0.15	147.6	147.6	147.6
0.50	195.2	0.20	167.3	167.3	167.3
0.67	(216.2)	0.55	185.0	(167.3)	185
0.70	(238.5)	1.10	(203.7)	(167.3)	(185)
0.71	(268.3)	1.15	(228.8)	(167.3)	(185)

^{A} $K_{Jc(1T)} = K_{Jc(0.5T)} - 20) \bullet (0.5 / 1)^{1/4} + 20$ Mpa$\sqrt{\text{m}}$

$$K_{o(1T)} = \left[\sum_{i=1}^{N} \frac{(K_{Jc(i)} - 20)^4}{r}\right]^{1/4} + 20 \tag{X1.6}$$

where:

N = 12,
r = 7,
$K_{o(1T)}$ = 186 MPa$\sqrt{\text{m}}$,
$K_{Jc(med)}$ = 171 MPa$\sqrt{\text{m}}$, and
T_o = 1°C.

X2. MASTER CURVE FIT TO DATA

X2.1 ***Select Test Temperature*** (see 8.4):
X2.1.1 Six 1/2T compact specimens,
X2.1.2 A 533 grade B base metal, and
X2.1.3 Test temperature, T = −75°C.
X2.2 In this data set, there are no censored data.

Rank (i)	$K_{Jc(1/2T)}$ (MPa$\sqrt{m}$)	$K_{Jc(1T)}$ Equivalent (MPa$\sqrt{m}$)
1	91.4	80.0
2	103.1	89.9
3	120.3	104.3
4	133.5	115.4
5	144.4	124.6
6	164.0	141.1

X2.3 *Determine K_o using* Eq. X1.2:

$$K_{o(1T)} = 115.8 \text{ MPa}\sqrt{\text{m}}, \text{ and}$$

$$K_{Jc(med)} = [\ln(2)]^{1/4} (K_o - 20) + 20 = 107.4 \text{ MPa}\sqrt{\text{m}}.$$

X2.4 *Position Master Curve*:

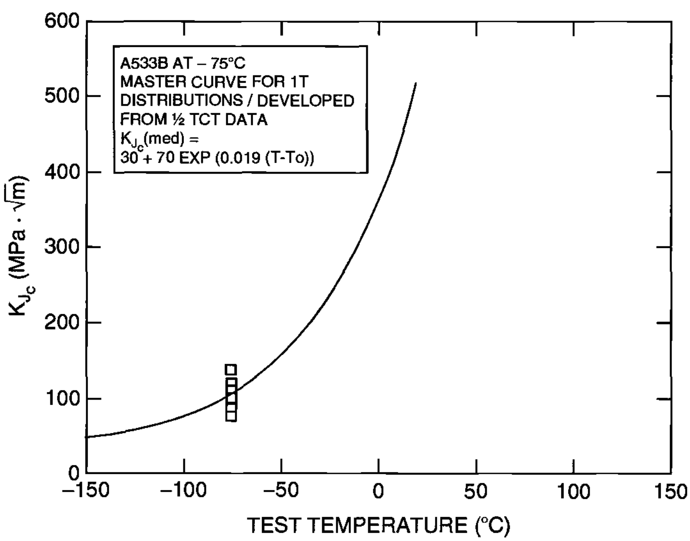
$$T_o = T - (0.019)^{-1} \ln[K_{Jc(med)} - 30)/70] \qquad \text{(X2.1)}$$

$$= -75 - ln[(108.5-30)/70]/0.019 = -80°\text{C}.$$

X2.5 *Master Curve*:

$$K_{Jc(med)} = 30 + 70 \exp[0.019(T + 80)] \qquad \text{(X2.2)}$$

X2.5.1 See Fig. X2.1.



Note 1—Toughness data are converted to 1T equivalence.

Fig. X2.1—Master Curve for 1T Specimens Based on 1/2 *T* Data Tabulated in Step X2.2

X3. EXAMPLE MULTITEMPERATURE T_o DETERMINATION

X3.1 *Material:*
A533 Grade B plate
Quenched and tempered
900°C WQ; and 440°C (5 h) temper

X3.2 *Mechanical Properties:*
Yield strength: 641 MPa (93 ksi)
Tensile strength: 870 MPa (117.5 ksi)
Charpy V:
28-J temperature = –5°C (23°F)
41-J temperature = 16°C (61°F)
NDT: 41°C (106°F)

X3.3 K_{Jc} *Limit Values:*
Specimen Types:
1/2T C(T) with $a_o/W = 0.5$
1T SE(B) with $a_o/W = 0.5$

Test Temperature (°C)	Yield Strength (MPa)	$K_{Jc(limit)}$ (MPa$\sqrt{m}$)	
		1/2T	1T
–10	651	239	338
–5	649	238	337
0	648	238	337
23	641	237	335

X3.4 *Slow-stable Crack Growth Limits:*

$$K_{Jc(1\ mm)} = 263 \text{ MPa } \sqrt{m} \text{ for 1T SE}(B) \textit{ specimen;}$$

$$K_{Jc(0.64\ mm)} = 255 \text{ MPa } \sqrt{m} \text{ for 1/2T C}(T) \textit{ specimen}$$

X3.5 *Estimation Procedure #1 from Charpy Curve:*

$$T_{o(est)} = T_{28J} + C = -5° - 18° = -23°C$$

$$T_{o(est)} = T_{41J} + C = 16° - 24° = -8°C$$

Conduct four 1T SE(B) tests at –20°C.

X3.6 T_o *Estimation Procedure #2 from Results of First Four Tests:*

First four tests at –20°C:

K_{Jc}, MPa$\sqrt{m}$
135.1
108.9
177.1
141.7

Calculate preliminary $T_{o(est)\#2}$ from data to determine allowable test temperature range:

$$K_{Jc(med)} = 137\ MPa\sqrt{m};$$

$$T_{o(est)\#2} = -42°C$$

Estimated temperature range or usable data:

$$= T_{o(est)\#2} \pm 50°C$$

$$= -92°C < T_i < +8°C$$

Now conduct additional testing within this range for T_o determination.
X3.7 *Calculation of T_o (Eq. 23):*
Use data between −92°C and 8°C based on $T_{o(est)\#2}$

$$T_o = -48°C$$

The valid test temeprature range is −98°C to 2°C. Original claculations were performed with data in this regime. Therefore, no iteration is required.

X3.8 *Qualified Data Summation:*

$(T - T_o)$ Range (°C)	Number of Valid Tests, r_i	Weight Factor, n_i	$r_i \cdot n_i$
50 to −14	43	1/6	7.2
−15 to −35	5	1/7	0.7
−36 to −50	0	1/8	0

Validity check:

$$\Sigma r_i n_i = 7.9 > 1.0$$

TABLE X3.1—Data tabulation.

Test Temperature, (°C)	Specimen		K_{Jc} (MPa$\sqrt{m}$)		δ_i
	Type	Size	Raw Data	1T Equivalent	
−130	C(T)	1/2T	59.5	53.2	1
			85.1	74.7	1
			55.3	49.7	1
			56.4	50.6	1
−80	C(T)	1/2T	51.3	46.3	1
			87.9	77.1	1
			113.4	98.5	1
−65	SE(B)	1T	73.9	73.9	1
			126.8	126.8	1
−55	C(T)	1/2T	167.7	144.2	1
			88.5	77.6	1
			115.2	100.0	1
			81.4	71.6	1

TABLE X3.1—Data Tabulation—*Continued*

Test Temperature, (°C)	Specimen		K_{Jc} (MPa$\sqrt{m}$)		δ_i
	Type	Size	Raw Data	1T Equivalent	
			121.9	105.7	1
			145.0	125.1	1
			104.2	90.8	1
			64.4	57.3	1
			96.8	84.6	1
			114.5	99.5	1
			107.4	93.5	1
			81.0	71.3	1
			70.0	62.0	1
			131.8	114.0	1
			69.5	61.6	1
			67.5	59.9	1
−30	C(T)	1/2T	102.3	89.2	1
			194.0	166.3	1
			170.4	146.5	1
			129.5	112.1	1
			118.2	102.6	1
			147.9	127.5	1
			178.8	153.5	1
			95.9	83.8	1
−20	SE(B)	1T	135.1	135.1	1
			108.9	108.9	1
			177.1	177.1	1
			141.7	141.7	1
			174.4	174.4	1
			84.8	84.8	1
			132.1	132.1	1
−10	C(T)	1/2T	211.4	180.9	1
			179.9	154.5	1
			171.8	147.6	1
			153.0	131.8	1
			236.9	(204)	0
			156.8	135	1
−5	C(T)	1/2T	121.5	105.3	1
			194.2	166.5	1
			110.4	96.0	1
			197.0	168.8	1
			134.7	116.5	1
			264.4	(203)	0
0	C(T)	1/2T	277.8	(198.9)	0
			218.9	187.2	1
			107.7	93.7	1
			269.3	(203)	0
			327.1	(203)	0
23	C(T)	1/2T	325 [A]	(202)	0
			328 [A]	(202)	0
			227	194	1

[A] R-curve (no cleavage instability).

X4. CALCULATION OF TOLERANCE BOUNDS

X4.1 The standard deviation of the fitted Weibull distribution is a mathematical function of Weibull slope, $K_{Jc(med)}$, and K_{min}, and because two of these are constant values, the standard deviation is easily determined. Specifically, with slope b of 4 and K_{min} = 20 MPa$\sqrt{m}$, standard deviation is defined by the following **(24)**:

$$\sigma = 0.28\ K_{Jc\ (med)}[1-20/K_{Jc(med)}] \qquad (X4.1)$$

X4.1.1 *Tolerance Bounds*—Both upper and lower tolerance bounds can be calculated using the following equation:

$$K_{Jc(0.xx)} = 20 + \left[ln\left(\frac{1}{1-0.xx}\right)\right]^{1/4} \{11 + 77\exp[0.019(T - To)]\} \qquad (X4.2)$$

where temperature "T" is the independent variable of the equation; xx represents the selected cumulative probability level; for example, for 2% tolerance bound, $0.xx$ = 0.02. As an example, the 5 and 95% bounds on the Appendix X2 master curve are:

$$K_{Jc(0.05)} = 25.2 + 36.6\ \exp\ [0.019\ (T + 80)] \qquad (X4.3)$$

$$K_{Jc(0.95)} = 34.5 + 101.3\ \exp\ [0.019\ (T + 80)]$$

X4.1.2 The potential error due to finite sample size can be considered, in terms of T_o, by calculating a margin adjustment, as described in X4.2.

X4.2 *Margin Adjustment*—The margin adjustment is an upward temperature shift of the tolerance bound curve, Eq. X4.3. Margin is added to cover the uncertainty in T_o that is associated with the use of only a few specimens to establish T_o. The standard deviation on the estimate on T_o is given by:

$$\sigma = \beta / \sqrt{r}\ (°C), \qquad (X4.4)$$

where:

r = total number of specimens used to establish the value of T_o.

X4.2.1 When $K_{Jc(med)}$ is equal to or greater than 83 MPa$\sqrt{m}$, β = 18°C **(25)**. If the 1T equivalent $K_{Jc(med)}$ is below 83 MPa$\sqrt{m}$, values of β must be increased according to the following schedule:

$K_{Jc(med)}$ **1*T* equivalent** [A] **(MPa$\sqrt{m}$)**	**β (°C)**
83 to 66	18.8
65 to 58	20.1

[A] Round off $K_{Jc(med)}$ to nearest whole number.

X4.2.2 To estimate the uncertainty in T_o, a standard two-tail normal deviate, Z, should be taken from statistical handbook tabulations. The selection of the con-

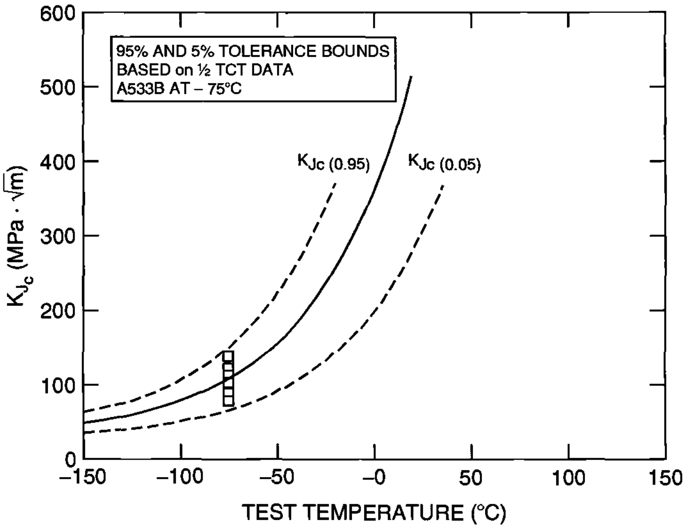


Fig. X4.1—Master Curve With Upper and Lower 95% Tolerance Bounds

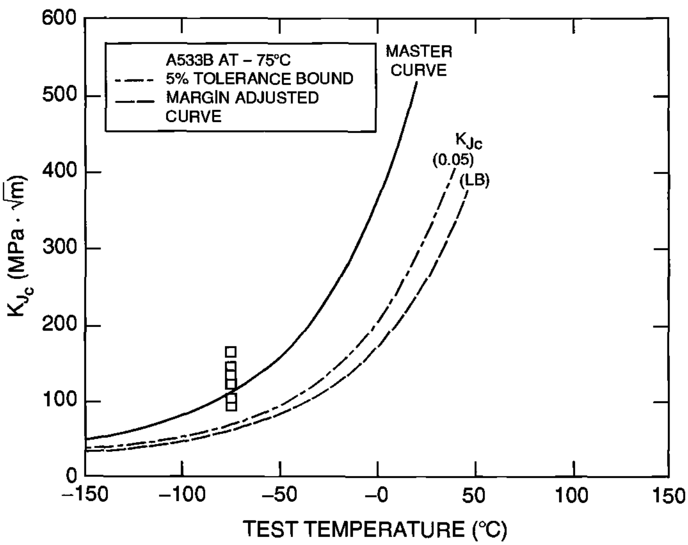


Fig. X4.2—Master Curve Showing the Difference Between 5% Tolerance Bound and Lower Bound That Includes 85% Confidence Margin on T_o

fidence limit for T_o adjustment is a matter for engineering judgment. The following example calculation is for 85% confidence (two-tail) adjustment to Eq. X4.3 for the six specimens used to determine T_o.

$$\Delta T_o = \sigma(Z_{85}) = \frac{18}{\sqrt{6}}(1.44) = 10°C \tag{X4.5}$$

$$T_o(margin) = T_o + \Delta T_o = -80°+10° = -70°C$$

Then the margin-adjusted 5% tolerance bound of Eq. X4.3 is revised to:

$$K_{Jc(05)} = 25.2 + 36.6 \exp\,[0.019(T + 70)] \tag{X4.6}$$

Eq. X4.6 is plotted in Fig. X4.2 as the dashed line (L.B.).