DISCUSSION

H. W. Liu[1]—Fracture mechanics encompasses an enormous body of knowledge, which includes fundamental theoies as well as practical experiments. It includes both the macroscopic phenomenological work as well as the microscopic mechanistic investigations. For their excellent appraisal, the authors should be complimented.

In this discussion, it is not intended to provide any new solution to fracture mechanics. It is rather intended to offer additional insight into the theoretical bases and the accepted practices in experimental investigations. With this understanding the direction of future research is clearly indicated.

The concept of fracture toughness, $\mathcal{G}_c$, can be derived from energy balance[2] as well as from the concept of stress and strain environments at the crack tip. The energy approach is well known and further elaboration is not necessary. An attempt will be made to bring forward the understanding of fracture mechanics from the concept of stress and strain environments. As noted in the appraisal, the stress-intensity factor, K, completely specifies the elastic stresses and strains in a region adjacent to the crack tip. It is well known that a plastic zone exists near the crack tip. It is not the elastic stresses and the elastic strains outside the plastic region that cause fracture. Rather, fracture results from the stresses and strains within the plastic zone. The elastic stresses are only a measure or an indicator of the stresses and strains within the plastic zone. The elastic stresses given by Eq 3 are approximate solutions, which are valid only in a region near the crack tip. The solid lines in Fig. 11 show the exact σ_x and σ_y in a cracked infinite plate along x-axis given by Inglis.[3] The dashed line is the approximate solution given by Eq 3. As the distance from the crack tip approaches zero, the approximate solution approaches the exact solution.

If the applied stresses in two specimens are $\bar{\sigma}_1$ and $\bar{\sigma}_2$ and the crack lengths are b_1 and b_2, respectively, and furthermore, for these two specimens, $K_1 = K_2$, according to Eq 3, the elastic stresses in these two specimens are identical. Figure 12 shows the ratio of σ_{y1} to σ_{y2} along the x-axis for different ratios of b_1/b_2. These curves were calculated from Inglis's exact solution. The figure indicates that near the crack tip the stresses are nearly equal to each other for various crack lengths. However, away from the crack tip, the stresses differ considerably even if K's are all the same. Therefore, it can be concluded that regions exist within which the stresses are approximately the same, if K's are the same. Let this region be prescribed by r' as shown in Fig. 13. If the plastic zone, r_p, is very

[1] Associate professor of metallurgy, Syracuse University, Syracuse. N. Y.

[2] A. A. Griffith, "The Phenomena of Rupture and Flow in Solids," *Philosophical Transactions*, Royal Society (London), Series A, Vol. 221, 1921.

G. R. Irwin, "Fracture Mechanics, Structural Mechanics," *Proceedings*, First Symposium on Naval Structural Mechanics, Pergamon Press, 1960.

H. W. Liu, "Fracture Criterion of Cracked Metallic Plate," *GALCIT SM 6329*, Graduate Aeronautical Labs, California Institute of Technology. July, 1963.

[3] E. E. Inglis, "Stresses in a Plate Due to the Presence of Cracks and Sharp Corners," *Transactions*, Institution of Naval Architects (London), Vol. 60, 1913, p. 219.

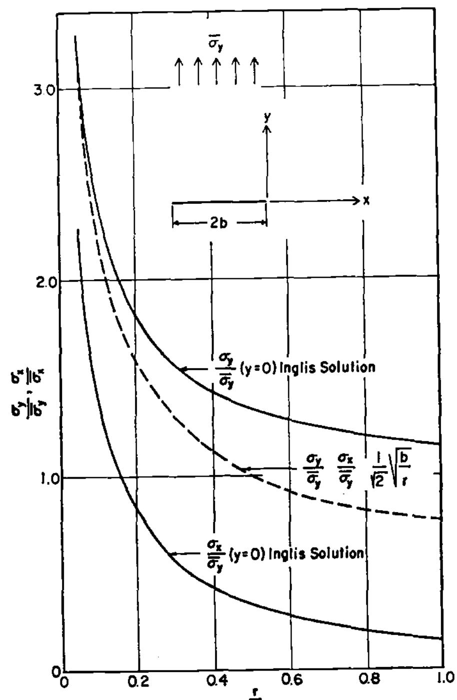

Fig. 11

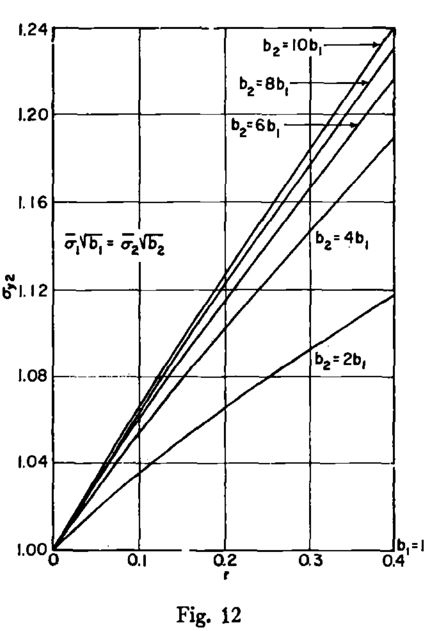

Fig. 12

small, that is, $r_p \ll r'$, the relaxation of stresses within the plastic region from that given by elastic solution will not change the stresses on r' significantly. Look at two regions bounded by r_1' and r_2' in two specimens. For these two specimens, $r_1' = r_2'$ and $K_1 = K_2$. Therefore, if the plastic zones are very small, the stresses on r_1' and r_2' are approximately the same. These two regions, bounded by r_1' and r_2', are geometrically identical and the applied stresses on the boundary are the same. Therefore, the stresses and strains at geometrically similar points, even within the plastic zone, are identical. Conse-

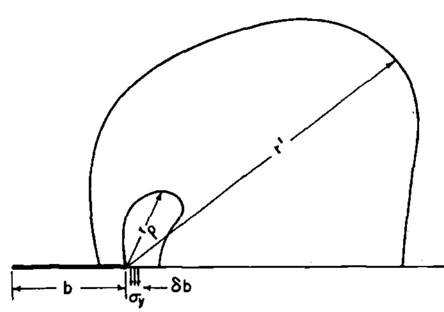

Fig. 13

quently, if one specimen fails at a stress and strain environment, so will the other at the same stress and strain environment. Therefore, it can be concluded that K_c for fracture is a constant; and $r_p \ll r'$ is a sufficient condition for a constant K_c. Small r_p implies low fracture stress and brittle mode of fracture.

If r_p is not small in comparison with r', the relaxation of the stresses in the plastic zone will change the stresses on r' significantly, so that the stress field of one crack tip interacts with the stress field of the other crack tip. For different crack lengths, the interaction is different. Therefore, the stresses on r' are no longer characterized by K. Hence K_c is no longer constant.

For a large plastic zone, in order to keep the condition of $r_p \ll r'$, the size of r' has to be enlarged. If r' is enlarged, Eq 3 will no longer give the correct stresses on r'. Figure 12 indicates that, in this case, σ_{y1} along the x-axis within the region r_1' is higher than σ_{y2} within the region r_2'. In order to give the same stress environment within r_1' and r_2', the applied stress on Specimen 2 has to be raised, or vice versa. Consequently, $K_{c1} < K_{c2}$, that is, K_c decreases with crack length. In this case, in order to maintain a constant K_c, an empirical correction factor is needed. This correction factor must be characterized by a small K_c increase for a long crack, and a considerable K_c increase for a short crack. Irwin's plastic-zone correction factor satisfies these requirements. The effective crack length, which is used to calculate K_{c1}, is the sum of the actual crack length plus the plastic-zone size, r_p. The correction factor, r_p, is more or less a constant. Therefore, for long cracks, that is, $2b \gg r_p$, the effect of the correction factor, r_p, is insignificant. On the other hand, for short cracks, the size of r_p relative to b increases; therefore, it increases the value of K_c considerably.

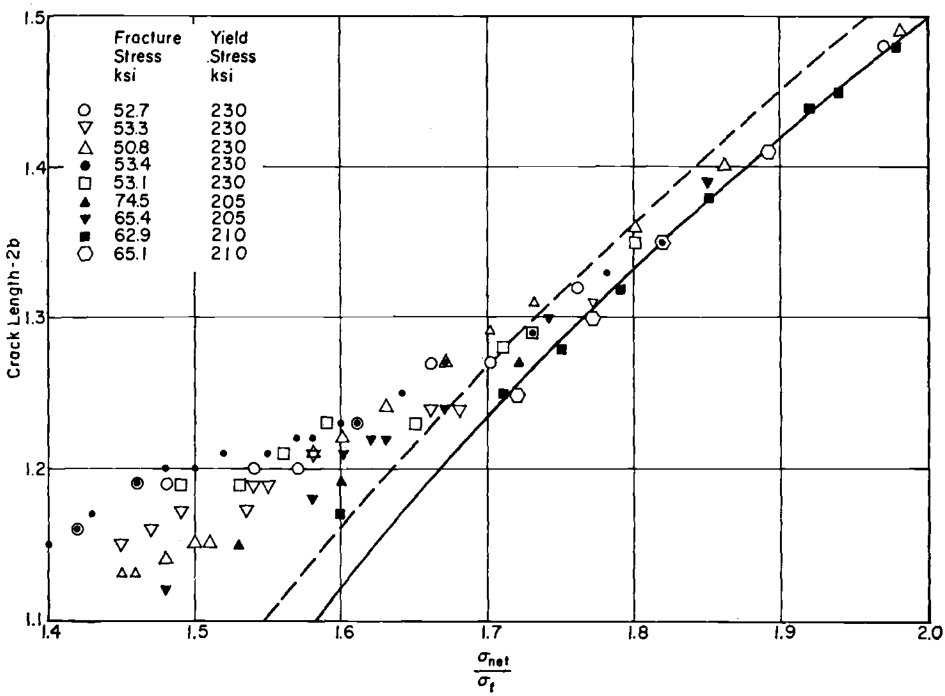

Fig. 14

The crack length is usually determined by either ink stain or visual observation of the "last unstable crack." This peculiar way of determining the crack length is another empirical correction factor. Figure 14 shows slow crack growth of centrally cracked 3-in. wide plates. The original fatigue cracks in the plate are 1 in. long. σ_f is the gross sectional fracture stress. As the load increases, the crack grows slowly. The solid line is the crack-growth line under the constant fracture load. The dashed line is the crack-growth line at 98 per cent of the fracture load. It is obvious that the crack growth at late stage is very unstable. The cracks grow with very little increase in load. For all practical purposes, the crack becomes very unstable at the length of 1.3 in., but the values used in calculating K_c are often 1.5 in

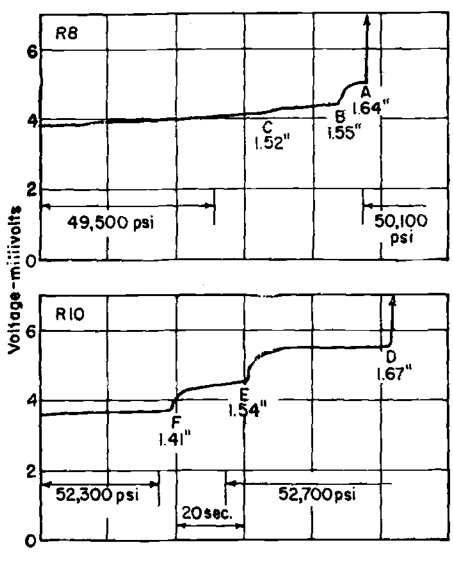

Fig. 15

or longer, if the last unstable crack length is used.

Figure 15 shows the slow crack growth of the same type of specimens of Fig. 14, as measured by voltage output.[4] Figure 15 also indicates the instability of the crack growth at late stage. The "last unstable crack" is 1.64 and 1.67 in. long in comparison with the original 1-in. fatigue crack. The extra "added length" serves as another correction factor, with the similar characteristics of r_p, in order to give constant K_c.

These corrective measures are needed if the size of the plastic zone is large, that is, for ductile fractures. They are needed because of our meager knowledge with respect to stresses and strains within the plastic zone. It has been noted that the stresses and strains within plastic zones depend upon the plastic behaviors of the materials such as strain-hardening exponent, etc.[5] Therefore it is uncertain that these two corrective measures can take care of both the plastic-zone size effect and the material effect. Consequently, this leads to the conclusion that an understanding of stresses and strains within plastic zones is the next logical step for further advances in fracture mechanics.

This discussion is a portion of Ref (3) and (7), which were written while the author was at the Graduate Aeronautical Laboratories of the California Institute of Technology. The experimental work was conducted at the H. F. Moore Fracture Research Laboratory at the University of Illinois. The assistance extended to the author by these two institutions is gratefully acknowledged.

A. KENT SHOEMAKER.[6]—One of the questions frequently raised in this paper was the effect of the notch-root radius on the crack toughness of a laboratory specimen. Although some work has been reported for high-strength steels

[4] H. W. Liu, "Effect of Water on the Fracture Strength of Specimens with a Central Notch," NRL Project 62R19-05, *Technical Memorandum No. 123*, U. S. Naval Research Labs., August, 1960.

[5] William W. Gerberich, "Plastic Strains and Energy Density in Cracked Plates. I. Experimental Techniques and Results," *GALCIT SM 63-23*, Graduate Aeronautical Labs., California Institute of Technology, June, 1963.

H. W. Liu, "Qualitative Discussion on the Effects of Strains Within Plastic Enclave on Fracture Criterion," *GALCIT SM 63-32*, Graduate Aeronautical Labs., California Institute of Technology, September, 1963.

[6] Department of Theoretical and Applied Mechanics, University of Illinois, Urbana, Ill.

by the ASTM committee reports,[7] there are very few data available for mild or "low-strength" steels which are temperature- and rate-sensitive.[8] The purpose of this discussion is to present some of these data for mild steel.

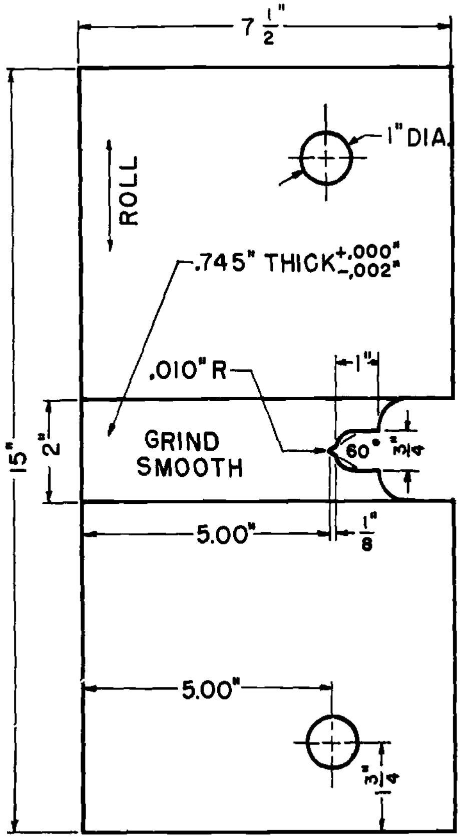

Fig. 16

In previous work, initial crack-extension (K^*_{Ic}) values were measured as a function of straining rate and temperature for $\frac{3}{4}$-in. thick A-201 mild steel[9] using a single-edge-notched specimen shown in Fig. 16. It was thought that the notch radius may have been too large to obtain minimum values of K^*_{Ic}. Subsequent experiments were made on specimens with notch radii varying from 0.0005 to 0.010 in. with essentially constant initial crack lengths and specimen geometry. Since no slow crack growth was observed in mild steel, the maximum load coincided with fracture initiation. These results for three different combinations of temperature and loading rate are shown in Fig. 17 in terms of the fracture load and the square root of the notch radii.

The notch radii of 0.0005, 0.001, 0.002, and 0.003 in. were fabricated by the use of a "string saw." A diamond abrasive compound was spread along the notch base and tungsten wires of the dimensions mentioned above were pulled back and forth across the notch base to make the desired radius. This sawing increased the crack length by an amount equivalent to three to four notch radii. The notch radii are quoted according to the radius of the wire used to cut them. The 0.005 and 0.010-in. radii were machined with a preshaped lathe tool mounted in a horizontal milling machine used in a fly-cutting manner.

The results, as shown in Fig. 17, indicated that at the low temperature, −270 F, the fracture load was approximately independent of the notch radius, while at −175 F the fracture load increased with increasing notch radius. There is enough scatter in the data, however, particularly at the lowest temperature and smallest notch radii,

[7] Fifth Report of the Special ASTM Committee, "Progress in Measuring Fracture Toughness and Using Fracture Mechanics," *Materials Research & Standards*, Vol. 4, No. 3, March, 1964, pp. 107–119.

[8] M. J. Manjoine, "Biaxial Brittle Fracture Tests," ASME *Paper No. 64-Met-3*, Am. Soc. Mechanical Engrs., 1964.

[9] A. K. Shoemaker, "The Influence of Temperature and Strain Rate on Crack Toughness of Mild Steel," *T&AM Report No. 235*, University of Illinois, Urbana, Ill., November, 1962.

to suggest a possible deviation from a straight-line relationship.

The trends can perhaps be explained by previous work[10] where it was found that for temperatures just below the transition range, equivalent to the −175 F data, fracture occurred after large numbers of microcracks had formed in the yielded zone at the crack tip. However, at still lower temperatures, equivalent to the −270 F data, few microcracks were found near the fracture initiation, thus indicating that cleavage fracture occurred from the first microcracks formed. Thus the independence of the fracture load with notch radius at −270 F is perhaps dependent upon a constant plastic-zone size necessary to form a microcrack. This very low-temperature cleavage fracture which initiated from the first formed microcracks would also indicate the possibility of greater data scatter at these temperatures; since the plastically deformed zone of material is very small, there is less probability of a random microcrack starting and growing in this smaller zone compared with a larger zone which occurs at a higher temperature. This is further exemplified by the two specimens which did not fracture at the very low temperature.

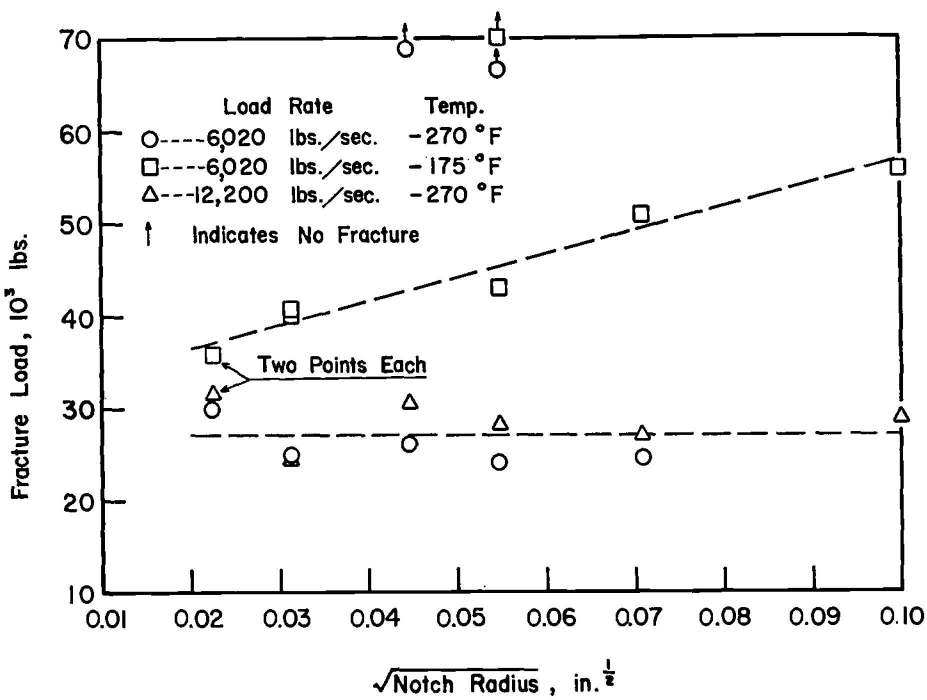

Fig. 17

The specimen which did not fracture at −175 F had not been cut by the string saw in the central section of the notch base. Thus the notch radius at the center section was somewhat in excess of 0.020 in.

[10] G. T. Hahn, W. S. Owen, B. L. Averbach, and M. Cohen, "Micromechanism of Brittle Fracture in a Low-Carbon Steel," *Welding Journal* (Research Supplement), Vol XXIV, No. 9, September, 1959, p. 367-s.

The data presented above are offered only as a preliminary study of the notch-sharpness effect on initial crack extension values for mild steel. The specimen configuration used did not lend itself to the fabrication of a natural crack so these data do not appear in Fig. 17. At the very low temperature, the possibility of a slight increase in fracture load as a natural crack tip is approached could be construed from the data. However, these data as well as the mechanism of fracture which cause them are far from conclusive and warrant further attention. It does appear that even though crack blunting occurs at the higher test temperature, a notch radius of not more than 0.0005 to 0.001 in. is necessary to approach minimum initial crack extension values for mild steel.

V. WEISS AND S. YUKAWA (*authors*).—The authors are grateful for the contributions of Prof. Liu and Mr. Shoemaker which serve to emphasize the importance of a better understanding of plasticity phenomena in the immediate vicinity of a stress raiser such as a notch or a crack. The data of Fig. 14, which show stable slow crack growth with increasing gross-section stress (load) clearly indicate the importance of a better understanding of the crack length–plastic zone size relationship for finite thickness and finite width specimens, and the complexity of the onset of crack instability in semi-brittle materials.

The microcrack explanation of the data in Fig. 17, as well as stress-state considerations, are probably applicable. The latter would suggest a varying stress biaxiality with varying root radius corresponding to the ratio of thickness to root radius. The plastic-zone size ahead of the notch is then determined not only by the value of the root radius but also by the corresponding stress state, that is, it would decrease on going from plane stress to plane strain.