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LETTER

Estimating the Tortuosity Factor

To the editor:

In the studies of the flow of fluid through a porous medium, such as a filter medium, the fluid flows a distance somewhat longer than the mere thickness of the medium. The ratio of path length to medium thickness is called the tortuosity factor.

A method is presented of estimating what that factor might be. The estimate follows what is intuitively obvious: the greater the porosity (that is, the ratio: void volume/bulk volume) the lower the tortuosity factor.

Imagine a plane cut through a filter medium perpendicular to the flow of fluid, and that this plane consists of a grid work. Consider further that the thickness of this plane equals the length of the side of an individual square on the grid. In addition, consider that this dimension approximates the thickness of a fiber or particle making up the structure of this filter medium. We thus see a plane of cubes, some empty and some occupied.

Now consider the flow of fluid through this grid. Consider N number of slugs of fluid, where each slug is small enough to pass through an empty cube in this grid. As these slugs approach the face of the grid N_p will pass straight through (where p is porosity). Another Nq (where q is $1 - p$) will hit a solid face and will have to take one side step looking for an empty cube. Of those Nq slugs that took one side step, Nqp found empty cubes and passed through. The remaining Nq^2p had to take a second side step still looking for an empty cube. As this sequence continues, we can tally up the number of side steps taken, as in Table 1.

For an example where $p = 0.8$ and $q = 0.2$ see Table 2.

TABLE 1—*Format for calculating number of side steps.*

(A) Number of Slugs	(B) Number of Side Steps Taken by Each Slug	(A) × (B)
Nq	1	Nq
Nq^2p	2	$2\,Nq^2p$
Nq^3p^2	3	$3\,Nq^3p^2$
Nq^4p^3	4	$4\,Nq^4p^3$
Total number of side steps: sum (A) × (B)		

TABLE 2—*Example of calculations using format of Table 1 where* p = *0.8,* q = *0.2.*.

(A)	(B)	(A) × (B)
N (0.2)	1	N 0.2000
N (0.04)(0.8)	2	N 0.064
N (0.008)(0.64)	3	N 0.0154
N (0.0016)(0.512)	4	N 0.00328
N (0.00032)(0.4096)	5	N 0.00065
Total		N 0.28918

TABLE 3—*Tortuosity factor* T *as a function of porosity* p.

p	T
0.4	2.03
0.5	1.88
0.6	1.65
0.7	1.48
0.8	1.29
0.9	1.12

Now, since all the slugs took one pass-through step in addition to the total of side steps in Table 2, the grand total of all steps is N 1.289 for N number of slugs. Thus, on the average the fluid traveled 1.289 times the thickness of the grid—or thickness of the filter medium.

The results of other calculations are shown in Table 3.

We would think that the present estimate holds for not only those filter media composed of a random packing of fibers or particles, but also for membranes produced by the solvent-cast, phase-inversion process, and for woven cloths.

Of course, the estimate does not hold for otherwise solid sheets containing straight-through holes, such as Nuclepore® membranes or photo-etched metal sheets. In such media the tortuosity factor is very close to 1.0.

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