LETTER

A Probe to Measure Friction Coefficient

To the editor:

The ingenious "tuning fork" penetrometer of Kolbe et al presented in the paper "A Probe to Measure Friction Coefficient of Cohesionless Soils," which appeared in your March/June 1982 issue (pp. 42-46), may offer some insights into the bearing capacity theory that in turn will offer some insights into the functioning and use of the device.

The tendency for the penetrometer face to spread under vertical load is pertinent to the rough-base bearing capacity theories of Terzaghi [1] and Meyerhoff [2]. Let us assume that lateral spreading force R on each semicircular face represents the sum of components of radial friction τ acting on elemental areas $rdrd\theta$

$$R = \tau \int_0^{180^\circ} \int_0^{r_1} r \cos\theta \, dr d\theta \tag{1}$$

where r_1 is the radius of the face. Integration gives $R = r_1^2 \tau$. A fully developed radial force R_R would be $R_R = (\pi/2)\, r_1^2 \tau$ giving a ratio $R_R/R = \pi/2 = 1.57$.

Next let us test if the radial face friction is fully developed in which case $R_{R\max}/N = \tan\phi$. If $\phi = 38^\circ$, $\tan\phi = 0.78$, whereas the authors' experimental regression gives $R_R/N = 0.133 \times \pi/2 = 0.21$ indicating that face friction is only partially developed. A similar result is obtained with other values of ϕ. The reduction in shear δ compared to ϕ is expected regardless of the degree of base roughness as shear cannot develop in the central zone because of opposing shear directions along the base (Fig. 1b).

The Terzaghi "rough-base" geometry shown in Fig. 1 assumes full development of base friction, and describes a central soil wedge with a base angle $\psi = \phi$. The ideal smooth-base geometry of Prandtl and others gives a deeper wedge with a base angle $\psi = 45 + \phi/2$ [3]. Meyerhoff [2] suggested intermediate base angles that give lowest calculated bearing capacities.

The base ψ angles (Fig. 1b) derive from the Mohr diagram of Fig. 1a. If base friction $\delta = 0$, the pole is at P; shear lines are oriented parallel to PT and PT′; and $\psi = 45 + \phi/2$ per the Prandtl geometry. If δ is the maximum equal to ϕ, the pole moves to T and shear lines are OT and TT′ giving $\psi = \phi$ as suggested by Terzaghi [1]. If δ is intermediate between 0 and ϕ, the active pole is at an intermediate position M with shear lines MT and MT′ and an intermediate value for ψ.

The probe R/N values times $\pi/2$ would appear to correspond to MQ/OR in Fig. 1a, OR representing the applied normal stress to initiate shear in the soil and MQ the developed shear stress on the face. By addition of angles at M in Fig. 1a it can be shown that $\angle TXO = 90^\circ - \phi$ and $\angle MXO = 90^\circ - 2\psi + \phi$. By writing their sines in terms of the radius of the Mohr circle, one obtains

$$\mathrm{MQ/TR} = \frac{\cos\phi}{\cos(2\psi - \phi)} \tag{2}$$

If we let n be the ratio MQ/TR of developed shear to maximum shear, solution of Eq 2 for ψ gives

$$\psi = 1/2\,[\cos^{-1}((1/n)\cos\phi) + \phi] \tag{3}$$

It also can be seen in Fig. 1a that OS = OQ − SQ = OR − SR. Then

$$\mathrm{MQ}(\cot\delta - \cot\psi) = \mathrm{TR}(\cot\phi - \cot\psi) \tag{4}$$

and

$$\delta = \cot^{-1}[1/n\,(\cot\phi - \cot\psi) + \cot\psi] \tag{5}$$

If we substitute selected authors' experimental regression values for $|\phi|$ and n, we obtain from Eqs 3 and 5

ϕ	38°	48°	57°
n	0.209	0.283	0.382
ψ	59.3°	63.5°	67.5°
δ	14.5°	27.5°	44.2°
δ/ϕ	0.381	0.573	0.775

It will be seen that as ϕ increases the relative δ/ϕ also increases, probably reflective of the greater tendency for volume expansion at higher ϕ angles caused by dilatancy. The indicated ψ values are 1.3 to 1.0 times the value of 1.2ϕ suggested by Meyerhoff, being closest for large ϕ angles. They also run about 0.92 times the maximum smooth-base value of $45 + \phi/2$ regardless of ϕ.

Thus, the measured values of R/N depend not only on ϕ but on δ and ψ as well. Since the three are interrelated, R/N does vary systematically with ϕ, but random variability in developed δ and in ψ nevertheless must contribute to experimental error.

Fig. 10 (p. 45) substantiates that there is significant data variability even with the use of averaged R/N values, and it would be instructive to show the ranges in R as well as averages. An outlying averaged data point at $\phi = 48^\circ$ gives an indicated $\phi = 55^\circ$, an error of 7° that would be explained by a change in ψ of 1.2° and in δ of 4.3°. The device thus appears to be more sensitive to changes in δ, and hence indirectly in density, than in ϕ. This also is shown by the regression values given above where the change of 19° in ϕ corresponds to a change of 30° in δ.

In Fig. 10 (p. 45) the justification for fitting a second-order equation is at best marginal because of the data scatter, and a linear regression would simplify calculation of the relevant confidence band for the prediction of individual values of ϕ from either individual or specifically averaged R/N measurements.

The split probe does present an advance over blunt-ended penetrometers by presenting two simultaneous measurements, MQ and OR in Fig. 1a, rather than simply OR. But as can be seen in the Fig. 1, MQ must depend on face friction δ and wedge angle ψ at least as much as it does on ϕ.

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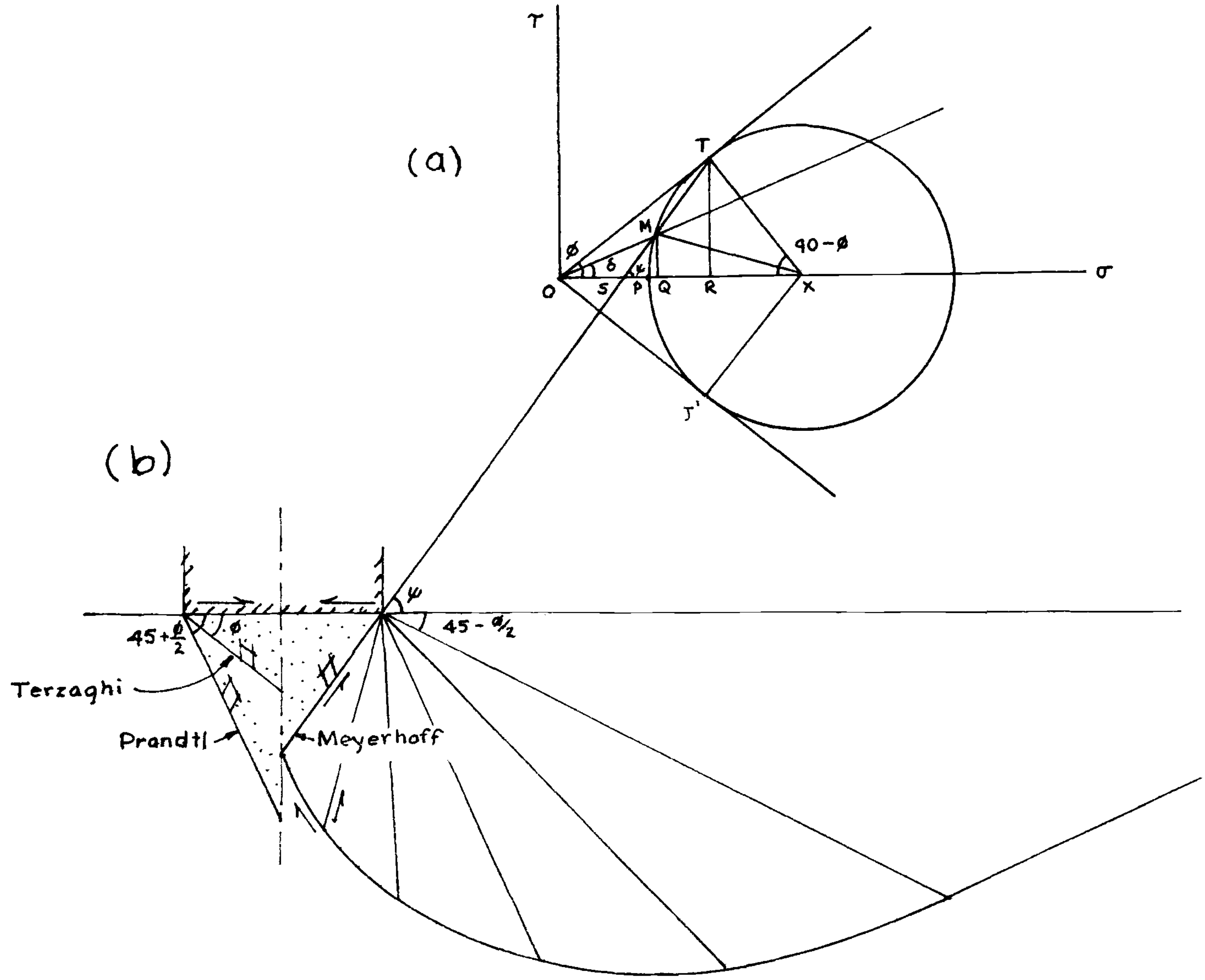

FIG. 1—(a) *Mohr diagram and* (b) *Terzaghi rough-base geometry, Prandtl ideal smooth-base geometry, and Meyerhoff suggested intermediate base angles.*

[1] Terzaghi, K., *Theoretical Soil Mechanics*, Wiley, New York, 1943.

[2] Meyerhof, G. G., "Influence of Roughness of Base and Groundwater Conditions on the Ultimate Bearing Capacity of Foundations," *Geotechnique*, Vol. 5, No. 3, Sept. 1955, pp. 227-242.

[3] Spangler, M. G. and Handy, R. L., *Soil Engineering*, 4th ed., New York, Harper and Row, 1982.

Author's reply

I appreciate Prof. Handy's interest and comments on the soil probe technical note. The use of the Mohr circle seems to offer another way to describe the interaction of stresses on the probe and to anticipate the influence of property variation on the measured forces. Because the Mohr circle represents stresses at a single point, I would raise a question as to how these relationships would hold when averaged over the entire face of the probe. The normal stress value σ certainly varies with radius or distance from center of a probe or footing. Shear stress τ and, therefore, the friction angle δ would vary as well. The forces measured by the probe are averaged from a radial distribution of stresses most of which would seem to be below the point of failure. The averaged friction angle δ would therefore also be based upon the same variation of stresses.

Prof. Handy's discussion following Eq 1 is not totally clear to me. It appears to relate the three-dimensional stress problem to a two-dimensional situation. But I wonder about the validity of assuming a constant stress τ in the integrals.

Our curve-fitting technique was based on a "best fit" scheme; as Prof. Handy suggests a linear equation might have served just as well. More careful preparation of specimens and (as is the usual case) more data would have improved the confidence. Also, relative error of radial force measurement in the loose sand specimens was high, implying a need for a more sensitive probe in some circumstances.

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