Literature Reviews

Environmental Effects on Composite Materials

Reviewed by K. L. Reifsnider, Virginia Polytechnic Institute and State University, Blacksburg, VA, coeditor of the Review.

REFERENCE: Springer, G. S., Ed., *Environmental Effects on Composite Materials*, Technomic, Westport, CT, 1981, 203 pages, $28.00.

This book is a compilation of articles written previously by the editor and his co-workers and published in the *Journal of Composite Materials*. Two chapters of new material have been included: one is a summary and the other provides numerical procedures and computer codes for calculating moisture diffusion through single and multilayered laminates. A subject index also appears. The editor suggests that the collection of articles should be "useful to engineers and scientists interested in the effects of moisture and temperature on composite materials." Topics discussed (article titles) include:

- Thermal Conductivities of Unidirectional Materials
- Moisture Absorption and Desorption of Composite Materials
- Moisture Absorption of Graphite-Epoxy Composition Immersed in Liquids and in Humid Air
- Moisture Absorption of Polyester-E Glass Composites
- Moisture Content of Composites Under Transient Conditions
- Effects of Moisture and Temperature on the Tensile Strength of Composite Materials
- Environmental Effects on the Elastic Moduli of Composite Materials
- Effects of Thermal Spiking on Graphite-Epoxy Composites
- Environmental Effects on Glass Fiber Reinforced Polyester and Vinyl-ester Composites
- Degradation of Tensile and Shear Properties of Composites Exposed to Fire or High Temperature
- Electrical Hazards Posed by Graphite Fibers
- Numerical Procedures for the Solution of One Dimensional Fickian Diffusion Problems

The style requirements of the journal have ensured a consistency of presentation. Each chapter contains an abstract, introduction, conclusion, and a generally systematic presentation with appropriate references. The book contains a modest index. The articles span a period from about 1968 to 1980.

As is evident from the topic list above, the most common "environment" considered by the articles in the book consists of temperature and moisture variations. The "effects" discussed include moisture absorption profiles (as a function of time and temperature); degradation of strength and elastic modulus caused by moisture and temperature increases under tensile and compressive loading; changes in strength and modulus (including shear strength and modulus) as a result of exposure to liquids such as saturated salt water, No. 2 diesel fuel, lubricating oil, antifreeze, and indolene; mass loss and changes in tensile and shear strength and stiffness as a result of exposure to fire; and the arcing voltage of graphite fibers settling on electrical conductors following their release from burned and unburned graphite-expoxy coupons.

The book's strongest feature is the experimental data presented in graphical and tabular form and the summaries of experimental data from other authors presented in tabular form. Hundreds of graphical characterizations of behavior for a wide range of materials and circumstances provide a valuable reference and a good basis for generalization of behavior. Chapter 10 alone has some 210 data plots. While the analysis presented is significant and the numerical codes listed are certainly of value to the user, it is much easier to find alternate analytical presentations than it is to find other sources of experimental data.

The analytical developments are based on Fickian diffusion concepts which do not allow for a variety of anomalous effects caused by such things as matrix cracking, voids, and so on. The treatments are essentially one-dimensional and can handle transient boundary conditions. A numerical analysis code is listed in Chapter 13 for the solution of such problems.

This book is a useful presentation of valuable information. The principal fault of the book is that articles tend to be presentations or expositions and not explanations in the textbook sense. However, some explanations can be found, and the extensive array of material property and behavior characterizations for an important set of environmental circumstances makes this collection of articles worthwhile for the engineer or scientist who has an interest in this area.

Developing Methods to Reduce Scatter of the Strength Properties of Advanced Composite Materials

Reviewed by K. L. Reifsnider, Virginia Polytechnic Institute and State University, Blacksburg, VA, coeditor of the Review.

REFERENCE: Brown, G. G., "Developing Methods to Reduce Scatter of the Strength Properties of Advanced Composite Materials," General Dynamics Report NADC-78078-60, Feb. 1980.

This report describes work aimed at fabricating composite laminates by methods that reduce the scatter of strength data, thereby effecting increases in the "A" and "B" design allowables for strength. The study was well organized, consisting of a first phase to develop baseline data, a second phase to develop fabrication schemes, and a third phase to compare data from laminates made using these techniques with the baseline data. Tensile, compressive, and rail shear data for unidirectional, ± 45, and $[\pm 45_2, 0_8, \mp 45_2]$ laminates were used for comparisons. The methods were defined as "optimum" on the basis of the compressive strength of unidirectional specimens. Plate specimens and specimens with center holes were tested.

Several prefabrication techniques were studied. In the first, called "stretch staging," the prepreg was loaded with a static load while stretched over a 1.2-m (4-ft) drum, then placed in an air circulating oven for staging (typically at 93°C [200°F] for up to 30 min). Flat laminates of the $[\pm 45_2, 0_8, \mp 45_2]$ type were then fabricated and tested. All compressive strengths were at least 10% above the average of the baseline data. The optimum stretch staging for the AS-3501-6 graphite/epoxy system used was 93°C (200°F) for 20 min with a weight of 302 N (68 lb) on the 150-mm

(6-in.) prepreg tape, which reduced the coefficient of variation significantly and increased strength modestly.

Another modification tested was to add chopped AS graphite fiber to the surface of each ply of the prepreg (about 15% by volume) to increase inter-ply strength. This addition degraded the transverse tensile strength of 0° unidirectional specimens and degraded the compressive strength of $[\pm 45_2, 0_8, \mp 45_2]$ laminates; however, adding 104 glass scrim cloth to the prepreg surfaces (from Hercules Corp.) in the laminates increased the transverse tensile strength and compressive strengths while reducing their coefficients of variation significantly.

The final method tested was "prestressing" by passing the prepreg over sets of small diameter rollers at cryogenic temperatures to fracture weak sites within the graphite filaments. Optimum conditions appeared to be −33.3°C (−25°F) with 9.4-mm (0.375-in.) diameter rollers.

Laminates made using the optimum techniques were then tested. Figures 1 and 2 show some of the positive results obtained. Figure 1 shows the largest effects, found in the ±45° laminate.

Figure 2 shows data for the 0° unidirectional laminates. For the three successful modifications tested, increases in transverse and compressive strength were more substantial than the reductions in scatter. Because of their cost, these additional methods (as the authors note) will probably be used only when a special need for

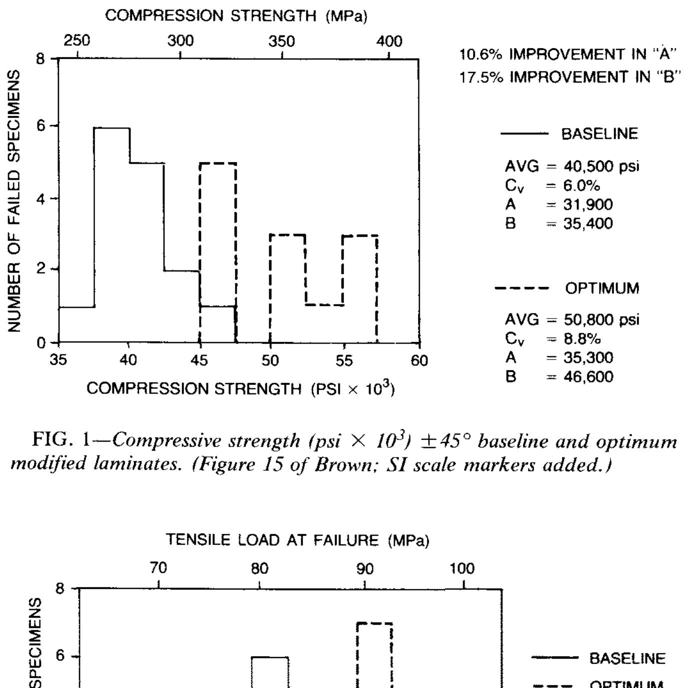

FIG. 1—*Compressive strength (psi × 10^3) ±45° baseline and optimum modified laminates. (Figure 15 of Brown; SI scale markers added.)*

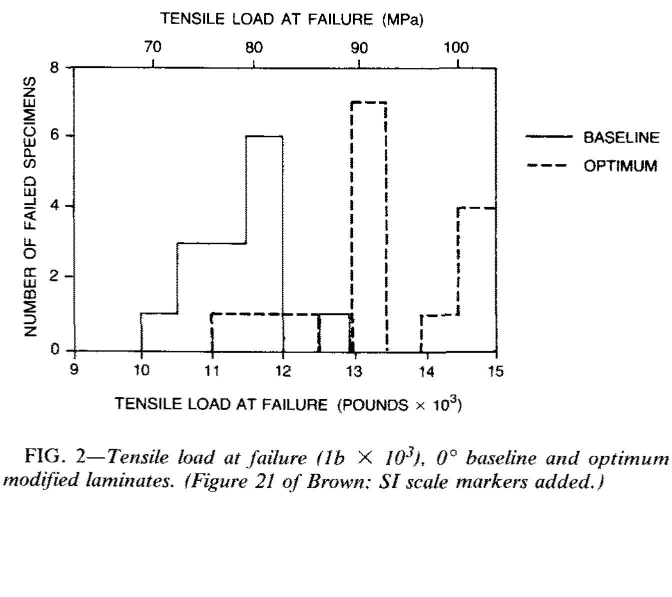

FIG. 2—*Tensile load at failure (lb × 10^3), 0° baseline and optimum modified laminates. (Figure 21 of Brown; SI scale markers added.)*

improved performance can justify the extra expenses. However, the work seems to verify some basic ideas about the effect of some mechanical processing variables on quasi-static properties and presents some useful and interesting data.

Analysis of Layered Composite Plates Accounting for Large Deflections and Transverse Shear Strains

Reviewed by K. L. Reifsnider, Virginia Polytechnic Institute and State University, Blacksburg, VA, coeditor of the Review.

REFERENCE: Reddy, J. N., "Analysis of Layered Plates Accounting for Large Deflections and Transverse Shear Strains," Virginia Polytechnic Institute College of Engineering Report VPI-E-81-12, April 1981.

Reddy's motivation for undertaking this study is the significant influence of transverse shear deformations on the quasi-static and dynamic response of thick composite plates. In his words, "The classical thin-plate theory (CPT) assumes that normals to the mid-surface before deformation remain straight and normal to the mid-surface after deformation, implying that thickness shear deformation effects are negligible. As a result, the natural frequencies, for example, calculated using the thin-plate theory are higher than those obtained by including the transverse shear deformation effects. Also, the transverse deflections predicted by the thin-plate theory are lower than those predicted by a shear deformable theory (SDT). Due to the low transverse shear modulus relative to the in-plane Young's moduli, the transverse shear deformation effects are even more pronounced in the composite plates. Thus a reliable prediction of the small deflection response characteristics of high modulus composite plates requires the use of shear deformable theories."

Furthermore, large deflections require additional rigor, primarily the inclusion of nonlinear terms in the equations of motion to account for midplane stretching caused by the interaction between membrane stresses and bending and shear plate curvatures.

This paper presents an excellent review of recent developments in the finite-element analysis of layered composite plates: the work of 100 authors is discussed, in all of which the effects of shear deformations and rotary inertia were *not* considered.

The paper presents a shear deformation theory that admits large, von Karmon-type rotations. The approach is based on the Yang-Norris-Stavsky (YNS) generalization of Reissner-Mindlin plate theory for homogeneous isotropic plates to arbitrary laminated anisotropic plates. To account for the midplane stretching caused by large deflections, the shear deformable theory of Whitney and Pagano [*1*] is modified to include large rotations.

The displacement field is assumed to be of the form

$$\begin{aligned} u_1(x, y, z, t) &= u(x, y, t) + z\,\psi_x(x, y, t) \\ u_2(x, y, z, t) &= v(x, y, t) + z\,\psi_y(x, y, t) \\ u_3(x, y, z, t) &= w(x, y, t) \end{aligned} \tag{1}$$

Here t is the time; u_1, u_2, and u_3 are the displacements in x, y, and z directions, respectively; u, v, and w are the associated midplane displacements; and ψ_x and ψ_y are the slopes in the xz and yz planes caused by bending only. Assuming that the plate is moderately thick and strains are much smaller than rotations, the nonlinear strain-displacement relations are expressed in the form,

$$\begin{aligned}
\epsilon_1 &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + z\frac{\partial \psi_x}{\partial x} \equiv \epsilon_1{}^0 + z\kappa_1 \\
\epsilon_2 &= \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + z\frac{\partial \psi_y}{\partial y} \equiv \epsilon_2{}^0 + z\kappa_2 \\
\epsilon_6 &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + z\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right) \equiv \epsilon_6{}^0 + z\kappa_6 \\
\epsilon_5 &= \psi_x + \frac{\partial w}{\partial x} \qquad \epsilon_4 = \psi_y + \frac{\partial w}{\partial y}
\end{aligned} \tag{2}$$

wherein the products of ψ_x, ψ_y, $\partial u_1/\partial x$, *and* $\partial u_2/\partial y$ are neglected. The author neglects body moments and surface shearing forces to write the equations of motion as

$$\begin{aligned}
N_{1,x} + N_{6,y} &= Pu_{tt} + R\psi_{x,tt} \\
N_{6,x} + N_{2,y} &= Pv_{tt} + R\psi_{y,tt} \\
Q_{1,x} + Q_{2,y} + N(N_i w) &= Pw_{tt} \\
M_{1,x} + M_{6,y} - Q_1 &= I\psi_{x,tt} + Ru_{tt} \\
M_{6,x} + M_{2,y} - Q_2 &= I\psi_{y,tt} + Rv_{tt}
\end{aligned} \tag{3}$$

Here P, R, and I are the normal, coupled normal-rotary, and rotary inertia coefficients; N_i, Q_i, and M_i are the stress and moment resultants; and $N(\ )$ is the nonlinear operator

$$\begin{aligned}
N(w,N_i) = N(w,N_i) &= \frac{\partial}{\partial x}\left(N_1\frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(N_6\frac{\partial w}{\partial x}\right) \\
&+ \frac{\partial}{\partial x}\left(N_6\frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial y}\left(N_2\frac{\partial w}{\partial y}\right)
\end{aligned} \tag{4}$$

The plate constitutive equations used are

$$\begin{aligned}
\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} &= \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ji} & D_{ij} \end{bmatrix} \begin{Bmatrix} \epsilon_j^0 \\ \kappa_j \end{Bmatrix} \\
\begin{Bmatrix} Q_2 \\ Q_1 \end{Bmatrix} &= \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \epsilon_4 \\ \epsilon_5 \end{Bmatrix}
\end{aligned} \tag{5}$$

based on plane stress assumptions. For the variational problem, the boundary conditions used are

$$\begin{aligned}
&\textit{essential:}\ \text{specify}, u_n, u_s, w, \psi_n, \psi_s \\
&\textit{natural:}\ \text{specify}, N_n, N_{ns}, q, M_n, M_{ns}
\end{aligned} \tag{6}$$

Reddy notes that these equations do not admit exact solutions. Instead, he presents a finite-element model based on an extension of the penalty plate-bending element he developed for the linear analysis of layered composite plates [*2*]. He states the weak form of Eq 3 over a typical element as

$$\begin{aligned}
0 = \int_{R^e} &[\delta u(Pu_{tt} + R\psi_{x,tt}) + \delta u_x N_1 + \delta u_y N_6 + \delta v(Pv_{tt} + R\psi_{y,tt}) \\
&+ \delta v_x N_6 + \delta v_y N_2 + \delta w(Pw_{tt}) + \delta w_x Q_1 + \delta w_y Q_2 \\
&+ \frac{\partial \delta w}{\partial x}\frac{\partial w}{\partial x}N_1 + \frac{\partial \delta w}{\partial y}\frac{\partial w}{\partial x}N_6 \\
&+ \frac{\partial \delta w}{\partial x}\frac{\partial w}{\partial y}N_6 + \frac{\partial \delta w}{\partial y}\frac{\partial w}{\partial y}N_2 \\
&+ \delta\psi_x(I\psi_{x,tt} + Ru_{tt}) + \delta\psi_{x,x}M_1 + \delta\psi_{x,y}M_6 + \delta\psi_x Q_1 \\
&+ \delta\psi_y(I\psi_{y,tt} + Rv_{tt}) + \delta\psi_{y,x}M_6 + \delta\psi_{y,y}M_2 + \delta\psi_y Q_2]\, dxdy \\
&+ \int_{C_n}(\delta u_n N_n + \delta u_s N_{ns})ds + \int_{C_q} \delta wq ds \\
&+ \int_{C_m}(\delta\psi_n M_n + \delta\psi_s M_{ns})ds
\end{aligned} \tag{7}$$

where N_n, N_{ns}, q, and M_n are plate boundary values (interior values cancel).

The author approximates u, v, w, and ψ_x over each element by

$$\begin{aligned}
u = U\tau(t), \qquad v = V\tau(t), \qquad w = W\lambda(t), \\
\psi_x = X\mu(t), \qquad \psi_y = Y\mu(t)
\end{aligned} \tag{8}$$

where U, V, and so on are given by

$$U = \sum_i^n \mathrm{U}_i \phi_i \tag{9}$$

where $\mathrm{U_i}$ is the value of U at node i; ϕ_i is the finite-element interpolation function at node i; n is the number of nodes in the element; and $\tau(t)$, $\lambda(t)$, and $\mu(t)$ are time-dependent functions whose specific form is to be determined. Substituting Eqs 8 and 9 into Eq 7, he obtains the following element equation:

$$[\mathbf{K}]\{\Delta\} + [\mathbf{M}]\{\Delta\} = \{\mathbf{F}\} \tag{10}$$

Here $\{\Delta\}$ is the column vector of the nodal values of the generalized displacements, $[\mathbf{K}]$ is the matrix of stiffness coefficients (which depends on the generalized displacements), $[\mathbf{M}]$ is the matrix of mass coefficients, and $\{\mathbf{F}\}$ is the column vector containing the boundary contributions. Reddy notes that the time functions are *not* harmonic; that is, strictly speaking, Eq 10 must be solved as a transient equation (even in the case of free vibrations). However, in the present analysis he assumes, for simplicity, that

$$\tau = \mu = \lambda^2 = \cos^2 \omega t \tag{11}$$

and retains only the first term of the cosine series. This assumption yields the standard eigenvalue problem in the case of natural vibration:

$$[(\mathbf{K}) - \omega^2(\mathbf{M})]\{\Delta\} = \{0\} \tag{12}$$

The solution procedure consists of a direct iteration, in which the global stiffness **[K]** is updated with the global displacement (eigenfunction) vector $\{\Delta\}$ from the previous iteration. $\{\Delta\}$ is set to zero at the beginning of the iteration procedure to obtain the linear solution (frequencies) of the problem at the end of the first iteration. The iteration is terminated when the nonlinear solution (frequencies) obtained during two consecutive iterations differ by some small number (say, 1%).

Reddy notes that the shear energy terms in the element matrices must be under integrated (a 1 by 1 Gauss rule used instead of a 2 by 2 rule for a four-node element, for example) to avoid "locking" (excessively stiff elements), a problem made especially severe by the large difference between the material's in-plane stiffness and shear stiffness.

To establish the effect of reduced integration and to illustrate the accuracy of the finite-element scheme, Reddy considers the quasi-static bending of several composite plates, using linear analysis, compared to results from the literature. The agreement is excellent. He also considers the nonlinear bending of isotropic and composite square plates under uniform loading and compares the stresses and deflections with other results from the literature. Numerical experiments were conducted to investigate the effect of element type and mesh on the results. The agreement with literature values is good, with some differences in edge stress values for certain cases. It is shown that shear deformation has a significant effect on the deflections of the thick plates considered. Extensive results and comparisons are presented.

Free vibrations of isotropic, orthotropic, and layered composite plates are then considered. Linear and nonlinear fundamental frequencies obtained by Reddy are compared with those available in the literature. Two example results follow.

Figure 3 shows the effect of orthotropy (E_1/E_2 for fixed $G_{12}/E_2 = 0.3846$, $\nu_{12} = 0.3$) on the linear fundamental frequencies λ_L and on the ratio of linear to nonlinear frequencies, $\gamma = \omega_L/\omega_{NL}$, of single-layer and two-layer cross-ply and angle-ply square plates with in-plane-to-thickness ratio $b/n = 10$.

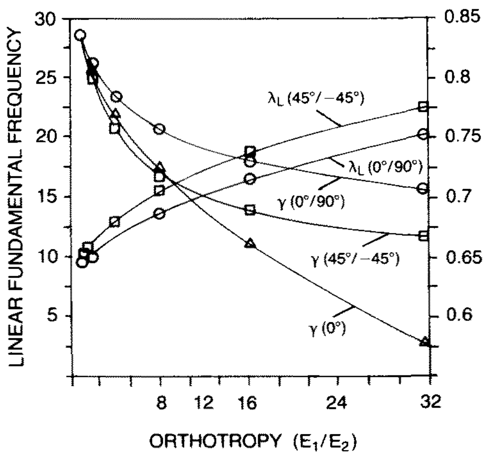

FIG. 3—*Effect of orthotropy* ($G_{12}/E_2 = 0.3846$, $\nu_{12} = 0.3$) *on the linear nondimensionalized fundamental frequency* λ *and on the ratio* ($\gamma = \omega_L/\omega_{NL}$) *of linear to nonlinear fundamental frequencies of square plates* ($b/h = 10$). (*Figure 15 of Reddy; slightly modified.*)

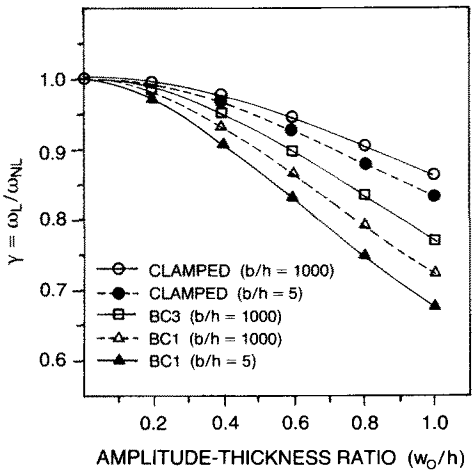

FIG. 4—*Ratio of linear to nonlinear fundamental frequencies* ($\gamma = \omega_L/\omega_{NL}$) *versus the amplitude-to-thickness ratio* (w_0/h) *for isotropic* ($\nu = 0.3$) *square plates (in-plane degrees of freedom not included). (Figure 14 of Reddy: slightly modified.)*

While the linear frequencies increase, the ratio of linear to nonlinear frequencies decreases with increasing E_1/E_2. Although the linear fundamental frequency of the angle-ply plate is larger than that of the cross-ply plate, the ratio ω_L/ω_{NL} is smaller for the angle-ply plate than for the cross-ply plate, indicating that the nonlinearity is more pronounced in the angle-ply plate than in the cross-ply plate.

Figure 4 shows the plot of frequency ratio versus the amplitude-to-thickness ratio for various boundary conditions and side-to-thickness ratio of isotropic ($\nu = 0.3$) square plates. Since the plate stiffness is proportional to the plate thickness and increases with edge constraints, the nonlinear frequencies are greater for thick plates than for thin plates.

Extensive additional results are presented, including the effects of boundary conditions on the variations identified.

Among other things, Reddy concludes that the four-node and nine-node isoparametric elements he uses (with reduced integration for shear energy terms) give accurate results while having the advantage of simplicity compared to the usual plate elements.

This report is a valuable addition to the literature (primarily the global aspects of response as noted by Reddy), especially in the context of the author's observation that "developments in computational mechanics related to finite-element analysis of plates and shells in the next decade will be largely concerned with the development of computationally simple elements that are capable of representing accurately physical features of the phenomena involved."

References

[1] Whitney, J. M. and Pagano, N. T., "Shear Deformation in Heterogeneous Anisotropic Plates," *Journal of Applied Mechanics*, Vol. 37, 1970, pp. 1031-1036.

[2] Reddy, J. N., "A Penalty Plate-Bending Element for the Analysis of Laminated Anisotropic Composite Plates," *International Journal of Numerical Methods Engineering*, Vol. 15, 1980, pp. 1187-1206.