Foreword

This publication, *Multiaxial Fatigue and Deformation: Testing and Prediction*, contains papers presented at the Symposium on Multiaxial Fatigue and Deformation: Testing and Prediction, which was held in Seattle, Washington during 19–20 May 1999. The Symposium was sponsored by the ASTM Committee E-8 on Fatigue and Fracture and its Subcommittee E08.05 on Cyclic Deformation and Fatigue Crack Formation. Sreeramesh Kalluri, Ohio Aerospace Institute, NASA Glenn Research Center at Lewis Field, and Peter J. Bonacuse, Vehicle Technology Directorate, U.S. Army Research Laboratory, NASA Glenn Research Center at Lewis Field, presided as symposium co-chairmen and both were editors of this publication.
# Contents

## Overview vii

### Multiaxial Strength of Materials

*Keynote Paper: Strength of a G-10 Composite Laminate Tube Under Multiaxial Loading*—D. SOCRÉ AND J. WANG 1

*Biaxial Strength Testing of Isotropic and Anisotropic Monoliths*—J. A. SALEM AND M. G. JENKINS 13

*In-Plane Biaxial Failure Surface of Cold-Rolled 304 Stainless Steel Sheets*—S. J. COVEY AND P. A. BARTOLOTTA 26

### Multiaxial Deformation of Materials

*Analysis of Characterization Methods for Inelastic Composite Material Deformation Under Multiaxial Stresses*—J. AHMAD, G. M. NEWAZ, AND T. NICHOLAS 41

*Deformation and Fracture of a Particulate MMC Under Nonradial Combined Loadings*—D. W. A. REES AND Y. H. J. AU 54

*Multiaxial Stress-Strain Notch Analysis*—A. BUCZYNSKI AND G. GLINKA 82

*Axial-Torsional Load Effects of Haynes 188 at 650° C*—C. J. LISSENDEN, M. A. WALKER, AND B. A. LERCH 99

*A Newton Algorithm for Solving Non-Linear Problems in Mechanics of Structures Under Complex Loading Histories*—M. ARZT, W. BROCKS, AND R. MOHR 126

### Fatigue Life Prediction Under Generic Multiaxial Loads

*A Numerical Approach for High-Cycle Fatigue Life Prediction with Multiaxial Loading*—M. DE FREITAS, B. LI, AND J. L. T. SANTOS 139

*Experiences with Lifetime Prediction Under Multiaxial Random Loading*—K. POTTER, F. YOUSEFI, AND H. ZENNER 157

*Generalization of Energy-Based Multiaxial Fatigue Criteria to Random Loading*—T. LAGODA AND E. MACHA 173

*Fatigue Strength of Welded Joints Under Multiaxial Loading: Comparison Between Experiments and Calculations*—M. WITT, F. YOUSEFI, AND H. ZENNER 191

### Fatigue Life Prediction Under Specific Multiaxial Loads

*The Effect of Periodic Overloads on Biaxial Fatigue of Normalized SAE 1045 Steel*—J. J. F. BONNEN AND T. H. TOPPER 213

*Fatigue of the Quenched and Tempered Steel 42CrMo4 (SAE 4140) Under Combined In- and Out-of-Phase Tension and Torsion*—G. LÖWISCH, H. BOMAS, AND P. MAYR 232

*In-Phase and Out-of-Phase Combined Bending-Torsion Fatigue of a Notched Specimen*—J. PARK AND D. V. NELSON 246
The Application of a Biaxial Isothermal Fatigue Model to Thermomechanical Loading for Austenitic Stainless Steel—S. Y. ZAMRIK AND M.L. RENAULD 266
Cumulative Axial and Torsional Fatigue: An Investigation of Load-Type Sequencing Effects—S. KALLURI AND P. J. BONACUSE 281

MULTIAXIAL FATIGUE LIFE AND CRACK GROWTH ESTIMATION

A New Multiaxial Fatigue Life and Crack Growth Rate Model for Various In-Phase and Out-of-Phase Strain Paths—A. VARVANI-FARAHANI AND T. H. TOPPER 305
Modeling of Short Crack Growth Under Biaxial Fatigue: Comparison Between Simulation and Experiment—H. A. SUHARTONO, K. PÖTTER, A. SCHRAM, AND H. ZENNER 323
Micro-Crack Growth Modes and Their Propagation Rate Under Multiaxial Low-Cycle Fatigue at High Temperature—N. ISOBE AND S. SAKURAI 340

MULTIAXIAL EXPERIMENTAL TECHNIQUES

An In-Plane Biaxial Contact Extensometer—O. L. KRAUSE AND P. A. BARTOLOTTA 369
Design of Specimens and Reusable Fixturing for Testing Advanced Aeropropulsion Materials Under In-Plane Biaxial Loading—J. R. ELLIS, G. S. SANDLASS, AND M. BAYYARI 382
Cruciform Specimens for In-Plane Biaxial Fracture, Deformation, and Fatigue Testing—C. DALLE DONNE, K.-H. TRAUTMANN, AND H. AMSTUTZ 405
Indexes 439
Overview

Engineering materials are subjected to multiaxial loading conditions routinely in aeronautical, astronautical, automotive, chemical, power generation, petroleum, and transportation industries. The extensive use of engineering materials over such a wide range of applications has generated extraordinary interest in the deformation behavior and fatigue durability of these materials under multiaxial loading conditions. Specifically, the technical areas of interest include strength of the materials under multiaxial loading conditions, multiaxial deformation and fatigue of materials, and development of multiaxial experimental capabilities to test materials under controlled prototypical loading conditions. During the last 18 years, the American Society for Testing and Materials (ASTM) has sponsored four symposia to address these technical areas and to disseminate the technical knowledge to the scientific community. Three previously sponsored symposia have yielded the following Special Technical Publications (STPs): (1) *Multiaxial Fatigue, ASTM STP 853*, (2) *Advances in Multiaxial Fatigue, ASTM STP 1191*, and (3) *Multiaxial Fatigue and Deformation Testing Techniques, ASTM STP 1280*. This STP is the result of the fourth ASTM symposium on the multiaxial fatigue and deformation aspects of engineering materials.

A symposium entitled “Multiaxial Fatigue and Deformation: Testing and Prediction” was sponsored by ASTM Committee E-8 on Fatigue and Fracture and its Subcommittee E08.05 on Cyclic Deformation and Fatigue Crack Formation. The symposium was held during 19–20 May 1999 in Seattle, Washington. The symposium’s focus was primarily on state-of-the-art multiaxial testing techniques and analytical methods for characterizing the fatigue and deformation behaviors of engineering materials. The objectives of the symposium were to foster interaction in the areas of multiaxial fatigue and deformation among researchers from academic institutions, industrial research and development establishments, and government laboratories and to disseminate recent developments in analytical modeling and experimental techniques. All except one of the 25 papers in this publication were presented at the symposium. Technical papers in this publication are broadly classified into the following six groups: (1) Multiaxial Strength of Materials, (2) Multiaxial Deformation of Materials, (3) Fatigue Life Prediction under Generic Multiaxial Loads, (4) Fatigue Life Prediction under Specific Multiaxial Loads, (5) Multiaxial Fatigue Life and Crack Growth Estimation, and (6) Multiaxial Experimental Techniques. This classification is intended to be neither exclusive nor all encompassing for the papers published in this publication. In fact, a few papers overlap two or more of the categories. A brief outline of the papers for each of the six groups is provided in the following sections.

Multiaxial Strength of Materials

Multiaxial strengths of metallic and composite materials are commonly investigated with either tubular or cruciform specimens. Three papers in this section address multiaxial strength characterization of materials. The first, and one of the two keynote papers in this publication, describes an experimental study on the strength and failure modes of woven glass fiber/epoxy matrix, laminated composite tubes under several combinations of tensile, compressive, torsional, internal pressure, and external pressure loads. This investigation illustrated the importance of failure modes in addition to the states of stress for determining the failure envelopes for tubular composite materials. The second paper describes a test rig for biaxial flexure strength testing of isotropic and anisotropic materials with the pressure-on-ring approach. The tangential and radial stresses generated in the disk specimens and the strains measured at failure in the experiments are compared with the theoretical predictions. The
third paper deals with in-plane biaxial testing of cruciform specimens manufactured from thin, cold-rolled, 304 stainless steel sheets. In particular the influence of texture, which occurs in the material from the rolling operation, on the effective failure stress is illustrated and some guidelines are proposed to minimize the rejection rates while forming the thin, cold-rolled, stainless steel into components.

**Multiaxial Deformation of Materials**

Constitutive relationships and deformation behavior of materials under multiaxial loading conditions are the subjects of investigation for the five papers in this section. The first paper documents detailed analyses of tests performed on off-axis tensile specimens and biaxially loaded cruciform specimens of unidirectional, fiber reinforced, metal matrix composites. The simplicity associated with the off-axis tensile tests to characterize the nonlinear stress-strain behavior of a unidirectional composite under biaxial stress states is illustrated. In addition, the role of theoretical models and biaxial cruciform tests for determining the nonlinear deformation behavior of composites under multiaxial stress states is discussed. Deformation and fracture behaviors of a particulate reinforced metal matrix alloy subjected to non-radial, axial-torsional, cyclic loading paths are described in the second paper. Even though the composite's flow behavior was qualitatively predicted with the application of classical kinematic hardening models to the matrix material, it is pointed out that additional refinements to the model are required to properly characterize the experimentally observed deformation behavior of the composite material. The third paper describes a methodology for calculating the notch tip stresses and strains in materials subjected to cyclic multiaxial loading paths. The Mroz-Garud cyclic plasticity model is used to simulate the stress-strain response of the material and a formulation based on the total distortional strain energy density is employed to estimate the elasto-plastic notch tip stresses and strains. The fourth paper contains experimental results on the elevated temperature flow behavior of a cobalt-base superalloy under both proportional and nonproportional axial and torsional loading paths. The database generated could eventually be used to validate viscoplastic models for predicting the multiaxial deformation behavior of the superalloy. Deformation behavior of a rotating turbine disk is analyzed with an internal variable model and a Newton algorithm in conjunction with a commercial finite element package in the fifth paper. Specifically, the inelastic stress-strain responses at the bore and the neck of the turbine disk and contour plots depicting the variation of hoop stress with the number of cycles are discussed.

**Fatigue Life Prediction under Generic Multiaxial Loads**

Estimation of fatigue life under general multiaxial loads has been a challenging task for many researchers over the last several decades. Four papers in this section address this topic. The first paper proposes a minimum circumscribed ellipse approach to calculate the effective shear stress amplitude and mean value for a complex multiaxial loading cycle. Multiaxial fatigue data with different waveforms, frequencies, out-of-phase conditions, and mean stresses are used to validate the proposed approach. Multiaxial fatigue life predictive capabilities of the integral and critical plane approaches are compared in the second paper for variable amplitude tests conducted under bending and torsion on smooth and notched specimens. Fatigue life predictions by the two approaches are compared with the experimental results for different types of multiaxial tests (pure bending with superimposed mean shear stress; pure torsion with superimposed mean tensile stress; and in-phase, 90° out-of-phase, and noncorrelated bending and torsional loads) and the integral approach has been determined to be better than the critical plane approach. In the third paper, a generalized energy-based criterion that considers both the shear and normal strain energy densities is presented for predicting fatigue life under multiaxial random loading. A successful application of the energy method to estimate the fatigue lives under uniaxial and biaxial nonproportional random loads is illustrated. Estimation of the fatigue lives of welded joints subjected to multiaxial loads is the subject of the fourth paper. Experimental results on flange-tube type welded joints subjected to cyclic bending and torsion are reported and
fatigue lifetime prediction software is used to calculate the fatigue lives under various multiaxial loading conditions.

**Fatigue Life Prediction under Specific Multiaxial Loads**

Biaxial and multiaxial fatigue and life estimation under combinations of cyclic loading conditions such as axial tension/compression, bending, and torsion are routinely investigated to address specific loading conditions. Five papers in this publication address such unique issues and evaluate appropriate life prediction methodologies. The effects of overloads on the fatigue lives of tubular specimens manufactured from normalized SAE 1045 steel are established in the first paper by performing a series of biaxial, in-phase, tension-torsion experiments at five different shear strain to axial strain ratios. The influence of periodic overloads on the endurance limit of the steel, variation of the crack initiation and propagation planes due to changes in the strain amplitudes and strain ratios, and evaluation of commonly used multiaxial damage parameters with the experimental data are reported. Combined in- and out-of-phase tension and torsion fatigue behavior of quenched and tempered SAE 4140 steel is the topic of investigation for the second paper. Cyclic softening of the material, orientation of cracks, and fatigue life estimation under in- and out-of-phase loading conditions, and calculation of fatigue limits in the normal stress and shear stress plane both with and without the consideration of residual stress state are reported. High cycle fatigue behavior of notched 1%Cr-Mo-V steel specimens tested under cyclic bending, torsion, and combined in- and out-of-phase bending and torsion is discussed in the third paper. Three multiaxial fatigue life prediction methods (a von Mises approach, a critical plane method, and an energy-based approach) are evaluated with the experimental data and surface crack growth behavior under the investigated loading conditions is reported. The fourth paper illustrates the development and application of a biaxial, thermomechanical, fatigue life prediction model to 316 stainless steel. The proposed life prediction model extends an isothermal biaxial fatigue model by introducing frequency and phase factors to address time dependent effects such as creep and oxidation and the effects of cycling under in- and out-of-phase thermomechanical conditions, respectively. Cumulative fatigue behavior of a wrought superalloy subjected to various single step sequences of axial and torsional loading conditions is investigated in the fifth paper. Both high/low load ordering and load-type sequencing effects are investigated and fatigue life predictive capabilities of Miner’s linear damage rule and the nonlinear damage curve approach are discussed.

**Multiaxial Fatigue Life and Crack Growth Estimation**

Monitoring crack growth under cyclic multiaxial loading conditions and determination of fatigue life can be cumbersome. In general, crack growth monitoring is only possible for certain specimen geometries and test setups. The first paper proposes a multiaxial fatigue parameter that is based on the normal and shear energies on the critical plane and discusses its application to several materials tested under various in- and out-of-phase axial and torsional strain paths. The parameter is also used to derive the range of an effective stress intensity factor that is subsequently used to successfully correlate the closure free crack growth rates under multiple biaxial loading conditions. The second paper on modelling of short crack growth behavior under biaxial fatigue received the Best Presented Paper Award at the symposium. The surface of a polycrystalline material is modeled as hexagonal grains with different crystallographic orientations and both shear (stage I) and normal (stage II) crack growth phases are simulated to determine crack propagation. Distributions of microcracks estimated with the model are compared with experimental results obtained for a ferritic steel and an aluminum alloy subjected various axial and torsional loads. Initiation of fatigue cracks and propagation rates of cracks developed under cyclic axial, torsional, and combined axial-torsional loading conditions are investigated for 316 stainless steel, 1Cr-Mo-V steel, and Hastelloy-X in the third paper. For each material, fatigue microcrack initiation mechanisms are identified and appropriate strain parameters to correlate the fatigue crack growth rates are discussed.
Multiaxial Experimental Techniques

State-of-the-art experimental methods and novel apparati are necessary to generate multiaxial deformation and fatigue data that are necessary to develop and verify both constitutive models for describing the flow behavior of materials and fatigue life estimation models. Five papers in this publication address test systems, extensometers, and design of test specimens and fixtures to facilitate multiaxial testing of engineering materials. The second of the two keynote papers reviews progress made in the design of multiaxial fatigue testing systems over the past five decades. Different types of loading schemes for tubular and planar specimens and the advantages and disadvantages associated with each of those schemes are summarized in the paper. Development of an extensometer system for conducting in-plane biaxial tests at elevated temperatures is described in the second paper. Details on the calibration and verification of the biaxial extensometer system and its operation under cyclic loading conditions at room temperature and static and cyclic loading conditions at elevated temperatures are discussed. Designing reusable fixtures and cruciform specimens for in-plane biaxial testing of advanced aerospace materials is the topic of investigation for the third paper. Feasibility of a fixture arrangement with slots and fingers to load the specimens and optimal specimen designs are established with finite element analyses. Details on three types of cruciform specimens used for biaxial studies involving fracture mechanics, yield surfaces, and fatigue of riveted joints are described in the fourth paper. Methods used for resolving potentially conflicting specimen design requirements such as uniform stress distribution within the test section and low cost of fabrication are discussed for the three types of specimens. The final paper describes the development and evaluation of a computer-controlled, electromechanical test system for characterizing mechanical behavior of composite materials under biaxial and triaxial loading conditions. Verification of the test system with uniaxial and biaxial tests on 6061-T6 aluminum, biaxial and triaxial test results generated on a carbon/epoxy cross-ply laminate, and proposed modifications to the test facility and specimen design to improve the consistency and accuracy of the experimental data are discussed.

The papers published in this book provide glimpses into the technical achievements in the areas of multiaxial fatigue and deformation behaviors of engineering materials. It is our sincere belief that the information contained in this book describes state-of-the-art advances in the field and will serve as an invaluable reference material. We would like to thank all the authors for their significant contributions and the reviewers for their critical reviews and constructive suggestions for the papers in this publication. We are grateful to the excellent support received from the staff at ASTM. In particular, we would like to express our gratitude to the following individuals: Ms. Dorothy Fitzpatrick, Ms. Hannah Sparks, and Ms. Helen Mahy for coordinating the symposium in Seattle, Washington; Ms. Monica Siperko for efficiently managing the reviews and revisions for all the papers; and Ms. Susan Sandler and Mr. David Jones for coordinating the compilation and publication of the STP.

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