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Manual

Presentation of Data and Control Chart Analysis 9th Edition

Stephen N. Luko, Editor





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Foreword

This *ASTM Manual on Presentation of Data and Control Chart Analysis* is the ninth edition of the *ASTM Manual on Presentation of Data* first published in 1933. This revision was prepared by the ASTM E11.30 Subcommittee on Statistical Quality Control, which serves the ASTM Committee E11 on Quality and Statistics.

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Preface

This *Manual on the Presentation of Data and Control Chart Analysis* (MNL 7) was prepared by ASTM's Committee E11 on Quality and Statistics to make available to the ASTM membership, and others, information regarding statistical and quality control methods and to make recommendations for their application in the engineering work of the Society. The quality control and other statistical methods considered herein are those methods that have been developed on a statistical basis to control the quality of product through the proper relation of specification, production, and inspection as parts of a continuing process.

The purposes for which the Society was founded—the promotion of knowledge of the materials of engineering and the standardization of specifications and the methods of testing—involve at every turn the collection, analysis, interpretation, and presentation of quantitative data. Such data form an important part of the source material used in arriving at new knowledge and in selecting standards of quality and methods of testing that are adequate, satisfactory, and economic, from the standpoints of the producer and the consumer.

Broadly, the three general objects of gathering engineering data are to discover: (1) physical constants and frequency distributions, (2) the relationships—both functional and statistical—between two or more variables, and (3) causes of observed phenomena. Under these general headings, the following more specific objectives in the work of ASTM may be cited: (a) to discover the distributions of quality characteristics of materials that serve as a basis for setting economic standards of quality, for comparing the relative merits of two or more materials for a particular use, for controlling quality at desired levels, and for predicting what variations in quality may be expected in subsequently produced material, and to discover the distributions of the errors of measurement for particular test methods, which serve as a basis for comparing the relative merits of two or more methods of testing, for specifying the precision and accuracy of standard tests and for setting up economical testing and sampling procedures; (b) to discover the relationship between two or more properties of a material, such as density and tensile strength; and (c) to discover physical causes of the behavior of materials under particular service conditions and the causes of nonconformance with specified standards in order to make possible the elimination of assignable causes and the attainment of economic control of quality.

Problems falling in these categories can be treated advantageously by the application of statistical methods and quality control methods. This Manual limits itself to several of the items mentioned under (a). **PART 1** discusses frequency distributions, simple statistical measures, and the presentation, in concise form, of the essential information contained in a single set of n observations. **PART 2** discusses the problem of expressing plus and minus limits of uncertainty for various statistical measures, together with some working rules for rounding-off observed results to an appropriate number of significant figures. **PART 3** discusses the control chart method for the analysis of observational data obtained from a series of samples and for detecting lack of statistical control of quality.

The present Manual is the ninth edition of earlier work on the subject. The original *ASTM Manual on Presentation of Data, STP 15*, issued in 1933, was prepared by a special committee of former Subcommittee IX on Interpretation and Presentation of Data of ASTM Committee E01 on Methods of Testing. In 1935, Supplement A on Presenting Plus and Minus Limits of Uncertainty of an Observed Average and Supplement B on “Control Chart” Method of Analysis and Presentation of Data were issued. These were combined with the original manual, and the whole, with minor modifications, was issued as a single volume in 1937. The personnel of the Manual Committee that undertook this early work were H. F. Dodge, W. C. Chancellor, J. T. McKenzie, R. F. Passano, H. G. Romig, R. T. Webster, and A. E. R. Westman. They were aided in their work by the ready cooperation of the Joint Committee on the Development of Applications of Statistics in Engineering and Manufacturing (sponsored by ASTM International and the American Society of Mechanical Engineers [ASME]) and especially of the chairman of the Joint Committee, W. A. Shewhart. The nomenclature and symbolism used in this early work were adopted in 1941 and 1942 in the American War Standards on Quality Control (Z1.1, Z1.2, and Z1.3) of the American Standards Association, and its Supplement B was reproduced as an appendix with one of these standards.

In 1946, ASTM Technical Committee E11 on Quality Control of Materials was established under the chairmanship of H. F. Dodge, and the Manual became its responsibility. A major revision was issued in 1951 as *ASTM Manual on Quality Control of Materials, STP 15C*. The Task Group that undertook the revision of

PART 1 consisted of R. F. Passano, Chairman; H. F. Dodge, A. C. Holman, and J. T. McKenzie. The same task group also revised **PART 2** (the old Supplement A), and the task group for revision of **PART 3** (the old Supplement B) consisted of A. E. R. Westman, Chairman; H. F. Dodge, A. I. Peterson, H. G. Romig, and L. E. Simon. In this 1951 revision, the term “confidence limits” was introduced, and constants for computing 95 % confidence limits were added to the constants for 90 % and 99 % confidence limits presented in prior printings. Separate treatment was given to control charts for “number of defectives,” “number of defects,” and “number of defects per unit,” and material on control charts for individuals was added. In subsequent editions, the term “defective” has been replaced by “nonconforming unit” and “defect” by “nonconformity” to agree with definitions adopted by the American Society for Quality Control in 1978. (See the American National Standard, ANSI/ASQC A1-1987, *Definitions, Symbols, Formulas and Tables for Control Charts*.)

There were more printings of *ASTM STP 15C*, one in 1956 and a second in 1960. The first added **E122**, *Recommended Practice for Choice of Sample Size to Estimate the Average Quality of a Lot or Process*, as an Appendix. This recommended practice had been prepared by a task group of ASTM Committee E11 consisting of A. G. Scroggie, Chairman; C. A. Bicking, W. E. Deming, H. F. Dodge, and S. B. Littauer. This Appendix was removed from that edition because it is revised more often than the main text of this Manual. The current version of **E122**, as well as of other relevant ASTM publications, may be procured from ASTM. (See the list of references at the back of this Manual.)

In the 1960 printing, a number of minor modifications were made by an ad hoc committee consisting of Harold Dodge, Chairman; Simon Collier, R. H. Ede, R. J. Hader, and E. G. Olds.

The principal change in *ASTM STP 15C* introduced in *ASTM STP 15D* was the redefinition of the sample standard deviation to be $s = \sqrt{\sum (X_i - \bar{X})^2 / (n-1)}$. This change required numerous changes throughout the Manual in mathematical equations and formulas, tables, and numerical illustrations. It also led to a sharpening of distinctions among sample values, universe values, and standard values that were not formerly deemed necessary.

New material added in *ASTM STP 15D* included the following items: The sample measure of kurtosis, g_2 , was introduced. This addition led to a revision of Table 1.8 and Section 1.34 of **PART 1**. In **PART 2**, a brief discussion of the determination of confidence limits for a universe standard deviation and a universe proportion was included. The Task Group responsible for this fourth revision of the Manual consisted of A. J. Duncan, Chairman; R. A. Freund, F. E. Grubbs, and D. C. McCune.

In the 22 years between the appearance of *ASTM STP 15D* and *Manual on Presentation of Data and Control Chart Analysis, 6th Edition*, there were two reprintings without significant changes. In that period, a number of misprints and minor inconsistencies were found in *ASTM STP 15D*. Among these were a few erroneous calculated values of control chart factors appearing in tables of **PART 3**. While all of these errors were small, the mere fact that they existed suggested a need to recalculate all tabled control chart factors. This task was carried out by A. T. A. Holden, a student at the Center for Quality and Applied Statistics at the

Rochester Institute of Technology, under the general guidance of Professor E. G. Schilling of Committee E11. The tabled values of control chart factors have been corrected where found in error. In addition, some ambiguities and inconsistencies between the text and the examples on attribute control charts have received attention.

A few changes were made to bring the Manual into better agreement with contemporary statistical notation and usage. The symbol μ (Greek “mu”) has replaced X (and \bar{X}) for the universe average of measurements (and of sample averages of those measurements). At the same time, the symbol σ has replaced σ' as the universe value of standard deviation. This entailed replacing σ with $s_{(rms)}$ to denote the sample root-mean-square deviation. Replacing the universe values p' , u' , and c' with Greek letters was thought to be worse than leaving them as they were. Section 1.33, **PART 1**, on distributional information conveyed by Chebyshev’s inequality, has been revised.

Summary of changes in definitions and notations.

MNL 7	STP 15D
μ, σ, p', u', c' (= universe values)	$\bar{X}, \sigma', p', u', c'$ (= universe values)
$\mu_{\sigma}, \sigma_{\sigma}, p_{\sigma}, u_{\sigma}, c_{\sigma}$ (= standard values)	$\bar{X}_{\sigma}, \sigma'_{\sigma}, p'_{\sigma}, u'_{\sigma}, c'_{\sigma}$ (= standard values)

In the twelve-year period since this Manual was revised again, three developments occurred that had an increasing impact on the presentation of data and control chart analysis. The first was the introduction of a variety of new tools of data analysis and presentation. The effect to date of these developments is not fully reflected in **PART 1** of this edition of the Manual, but an example of the “stem and leaf” diagram is now presented in Section 1.14. The *Manual on Presentation of Data and Control Chart Analysis* from the beginning has embraced the idea that the control chart is an all-important tool for data analysis and presentation. To integrate properly the discussion of this established tool with the newer ones presents a challenge beyond the scope of this revision.

The second development of recent years strongly affecting the presentation of data and control chart analysis is the greatly increased capacity, speed, and availability of personal computers and sophisticated hand calculators. The computer revolution has not only enhanced capabilities for data analysis and presentation but also enabled techniques of high-speed real-time data collection, analysis, and process control, which years ago would have been unfeasible, if not unthinkable. This has made it desirable to include some discussion of practical approximations for control chart factors for rapid, if not real-time, application. Supplement 3.A has been considerably revised as a result. (The issue of approximations was raised by Professor A. L. Sweet of Purdue University.) The approximations presented in this Manual presume the computational ability to take squares and square roots of rational numbers without using tables. Accordingly, the Table of Squares and Square Roots that appeared as an Appendix to *ASTM STP 15D* was removed from the previous revision. Further discussion of

approximations appears in Notes 8 and 9 of Supplement 3.B, **PART 3**. Some of the approximations presented in **PART 3** appear to be new and assume mathematical forms suggested in part by unpublished work of Dr. D. L. Jagerman of AT&T Bell Laboratories on the ratio of gamma functions with near arguments.

The third development has been the refinement of alternative forms of the control chart, especially the exponentially weighted moving average chart and the cumulative sum (“cusum”) chart. Unfortunately, time was lacking to include discussion of these developments in the fifth revision, although references were given. The assistance of S. J. Amster of AT&T Bell Laboratories in providing recent references to these developments is gratefully acknowledged.

Manual on Presentation of Data and Control Chart Analysis, 6th Edition, by Committee E11 was initiated by M. G. Natrella with the help of comments from A. Bloomberg, J. T. Bygott, B. A. Drew, R. A. Freund, E. H. Jebe, B. H. Levine, D. C. McCune, R. C. Paule, R. F. Potthoff, E. G. Schilling, and R. R. Stone. The revision was completed by R. B. Murphy and R. R. Stone with further comments from A. J. Duncan, R. A. Freund, J. H. Hooper, E. H. Jebe, and T. D. Murphy.

Manual on Presentation of Data and Control Chart Analysis, 7th Edition, was directed at bringing the discussions around the various methods covered in **PART 1** up to date, especially in the areas of whole number frequency distributions, empirical percentiles, and order statistics. As an example, an extension of the stem-and-leaf diagram was added that was termed an “ordered stem-and-leaf,” which made it easier to locate the quartiles of the distribution. These quartiles, along with the maximum and minimum values, were then used in the construction of a box plot.

In **PART 3**, additional material has been included to discuss the idea of risk, namely, the alpha (α) and beta (β) risks involved in the decision-making process based on data and tests for assessing evidence of nonrandom behavior in process control charts.

Also, use of the $s_{(rms)}$ statistic has been minimized in favor of the sample standard deviation s to reduce confusion as to their use.

Furthermore, the graphics and tables throughout the text have been repositioned so that they appear more closely to their discussion in the text.

Manual on Presentation of Data and Control Chart Analysis, 7th Edition, by Committee E11 was initiated and led by Dean V. Neubauer, Chairman of the E11.10 Subcommittee on Sampling and Data Analysis, which oversees this document. Additional comments from Stephen Luko, Charles Proctor, Paul Selden, Greg Gould, Frank Sinibaldi, Ray Mignogna, Neil Ullman, Thomas D. Murphy, and R. B. Murphy were instrumental in the vast majority of the revisions made in the sixth revision.

Manual on Presentation of Data and Control Chart Analysis, 8th Edition, had some new material in **PART 1**. The discussion of the construction of a box plot was supplemented with some definitions to improve clarity, and new sections were added on probability plots and transformations.

The eighth edition contained a new **PART 4** section that included material on measurement systems analysis, process capability, and process performance. This important section was deemed necessary because it is important that the measurement process be evaluated before any analysis of the process is begun. Work on the eighth edition was initiated and led by Dean V. Neubauer, chairman of the E11.30 Subcommittee on Statistical Quality Control, which oversees this document. Additional material from Stephen Luko, Charles Proctor, and Bob Sichi, including reviewer comments from Thomas D. Murphy, Neil Ullman, and Frank Sinibaldi, were critical to the vast majority of the revisions made in the seventh revision. Thanks must also be given to Kathy Dernoga and Monica Siperko of ASTM International Publications Department for their efforts in the publication of that edition.

The ninth edition of the Manual builds on earlier editions with additional new material in **PART 2** and **PART 4** as well as extensive editing. The project was initiated and led by Stephen N. Luko with additional material supplied by Jennifer Brown and S. Luko. Extensive reviews were provided by John Carson, Palona Carson, T. D. Murphy, Neil Ullman, Peter Fortini, and Jennifer Brown.

