

ASTM MANUAL  
*on*  
QUALITY CONTROL  
OF MATERIALS



*Prepared by*  
ASTM COMMITTEE E-11  
On Quality Control of Materials

- Part 1—Presentation of Data  
Part 2—Presenting  $\pm$  Limits of Uncertainty  
of an Observed Average  
Part 3—Control Chart Method of Analysis and Presentation  
of Data

*Special Technical Publication 15-C*

January, 1951

*Published by the*  
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# ASTM

*Purpose.—The promotion of knowledge of the materials of engineering, and the standardization of specifications and the methods of testing.*

THE American Society for Testing and Materials is a nonprofit, national educational, scientific, and technical society, whose purpose stated above may be summarized as "Research and Standards for Materials."

The research is effected through investigations by committees and by individual and company members of the Society, and by joint research projects with other organizations, the results of which are presented as reports and technical papers at Society meetings, and subsequently published. ASTM committees now have more than 100 research projects under way.

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Reg. U. S. Pat. Off.

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## PREFACE

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This Manual on the Quality Control of Materials was prepared by ASTM Technical Committee E-11 on Quality Control of Materials to make available to the ASTM membership, and others, information regarding statistical methods and quality control methods and to make recommendations for their application in engineering work of the Society. The quality control methods considered herein are those methods that have been developed on a statistical basis to control the quality of product through the proper relation of specification, production, and inspection as parts of a continuing process.

This Manual consists of three Parts dealing particularly with the analysis and presentation of data. It constitutes a revision and a replacement of the ASTM Manual on Presentation of Data whose main section and two supplements were first published respectively in 1933 and 1935. This early work was done with the ready cooperation of the Joint Committee on the Development of Applications of Statistics in Engineering and Manufacturing (sponsored by the American Society for Testing Materials and the American Society of Mechanical Engineers) and especially of the Chairman of the Joint Committee, W. A. Shewhart. Over the past 15 years this material has gone through a number of minor modifications and reprintings and has become a standard of reference over wide areas in both industrial and academic fields. Its nomenclature and symbolism were adopted in 1941 and 1942 in the American War Standards on Quality Control (Z1.1, Z1.2 and Z1.3) of the American Standards Association, and its Supplement B was reproduced as an appendix with one of these Standards.

The purposes for which the Society was founded—the promotion of knowledge of the materials of engineering, and the standardization of specifications and the methods of testing—involve at every turn the collection, analysis, interpretation and presentation of quantitative data. Such data form an important part of the source material used in arriving at new knowledge, and in selecting standards of quality and methods of testing that are adequate, satisfactory, and economic, from the standpoints of the producer and the consumer.

Broadly, the three general objects of gathering engineering data are to discover: (1) physical constants and frequency distributions, (2) relationships—both functional and statistical—between two or more variables, and (3) causes of observed phenomena. Under these general headings, the following more specific objectives in the work of the American Society for Testing Materials may be cited:

(a) to discover the distributions of quality characteristics of materials which serve as a basis for setting economic standards of quality, for comparing the relative merits of two or more materials for a particular use, for controlling quality at desired levels, for predicting what variations in quality may be expected in subsequently produced material; to discover the distributions of the errors of measurement for particular test methods, which serve as a basis for comparing the relative merits of two or more methods of testing, for specifying the precision and accuracy of standard tests, for setting up economical testing and sampling procedures;

(b) to discover the relationship between two or more properties of a material, such as density and tensile strength; and

(c) to discover physical causes of the behavior of materials under particular service conditions; to discover the causes of nonconformance with specified standards in order to make possible the elimination of assignable causes and the attainment of economic control of quality.

Problems falling in the above categories can be treated advantageously by the application of statistical methods and quality control methods. The present Manual limits itself to several of the items mentioned under (a) above. Part 1 discusses frequency distributions, simple statistical measures, and the presentation, in concise form, of the essential information contained in a single set of  $n$  observations. Part 2 discusses the problem of expressing  $\pm$  limits of uncertainty of an observed average of a single set of  $n$  observations, together with some working rules for rounding-off observed results to an appropriate number of significant figures. Part 3 discusses the control chart method for the analysis of observational data obtained from a series of samples, and for detecting lack of statistical control of quality.

This Manual is the first major revision of the earlier work. The original Manual and the two supplements were prepared by the Manual Committee of the former Subcommittee IX on Interpretation and Presentation of Data, of Committee E-1 on Methods of Testing. The personnel of the Manual Committee was as follows: Messrs. H. F. Dodge, chairman (1932-46), W. C. Chancellor (1934-37), J. T. MacKenzie (1932-46), R. F. Passano (1939-46), H. G. Romig (1938-46), R. T. Webster (1932-44), A. E. R. Westman (1932-34). Changes and additions have been made in line with comments and suggestions received from many sources. Since the last modification of the earlier work, the American Society for Quality Control has been organized (1946) and has assumed a responsible and recognized position in the field of quality control. Its cooperation in the present revision is hereby acknowledged.

The list of members of Committee E-11 appearing in this Manual shows the personnel of the committee as of the date of publication. During the preparation of the three parts of the Manual the following were also active members of the committee: Messrs. C. W. Churchman, H. F. Hebley, J. C. Hintermaier, R. F. Passano, A. I. Peterson, T. S. Taylor, John Tucker, Jr.

Additional subject material is under consideration by the committee for inclusion in this Manual as additional Parts.

January, 1951.

In this fifth printing of the Manual there has been included in the Appendix the Tentative Recommended Practice for Choice of Sample Size to Estimate the Average Quality of a Lot or Process (ASTM Designation: E 122). This recommended practice was prepared by Dr. W. Edwards Deming and Miss Mary N. Torrey and represents in part work done by Task Group No. 6 of Committee E-11, which consists of A. G. Scroggie, chairman, C. A. Bicking, W. Edwards Deming, H. F. Dodge, and S. B. Littauer.

September, 1956.

In this sixth printing of the Manual corrections have been made of the typographical errors on pp. 61, 62, 65, and 69.

December, 1957.

This seventh printing of the Manual includes several minor additions and revisions. The changes in Part 1 include revised values in Tables I (*c*) and II (*c*) (and corresponding values elsewhere in the Manual where referred to); also an addition to Section 4. Sections 20, 21, and 28 were modified to include formulas for  $s$  and  $s^2$ . In Part 3, Section 7 was expanded, and in the Example Sections 31, 32, and 33 the paragraph on Results was revised in Examples 2, 3, 4, 8, 13, 16, 21, and 22. The Appendix was expanded to include a List of Some Related Publications on Quality Control and Statistics and a Table giving a comparison of the symbols used in the Manual and those used in statistical texts. These changes were prepared by an Ad Hoc Committee on Modification of ASTM Manual. The personnel of this committee is as follows: H. F. Dodge, chairman, Simon Collier, R. H. Ede, R. J. Hader, and E. G. Olds.

July, 1960.

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<sup>a</sup> Available as a separate reprint from ASTM Headquarters.

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**T**HIS PUBLICATION is one of many issued by the American Society for Testing Materials in connection with its work of promoting knowledge of the properties of materials and developing standard specifications and tests for materials. Much of the data result from the voluntary contributions of many of the country's leading technical authorities from industry, scientific agencies, and government.

Over the years the Society has published many technical symposiums, reports, and special books. These may consist of a series of technical papers, reports by the ASTM technical committees, or compilations of data developed in special Society groups with many organizations cooperating. A list of ASTM publications and information on the work of the Society will be furnished on request.